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Numerical Solution of Singular Boundary Value Problems UsingGenetic Algorithm

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Abstract: *In this paper, we proposed expanding the application of genetic algorithm for solving some nonlinear singular boundary value problems arising in physiology applications which it is difficult to solve this type of problems because of a singularity at the boundary point x= 0. Genetic Algorithm is one of the evolutionary algorithms that uses principles inspired by natural population genetics to evolve solutions to problems and mimics Darwin's theory of biological evolution, whereby the fitness function (error function) is represented by the sum of mean square errors of the nonlinear differential equation and its boundary conditions. The approximate solution can be expressed as a polynomial of n degree, then the genetic algorithm is used to find the optimal coefficients of a polynomial that minimize the fitness function. Numerical examples are given to show the efficiency and accuracy of the proposed algorithm, our results are also compared with other existing results obtained by different methods such as Taylor's series method, homotopy analysis method and Adomian decomposition method.*

Keywords: *Evolutionary algorithms, Fitness Function, Genetic algorithm, Optimization problem, Physiology applications, Singularboundary value problems.*

INTRODUCTION

In recent years, singular boundary value problems (BVPs) have been applied in various fields of applied mathematics, physics, chemistry, and others, such as heat distribution in a human head, thermal explosions, chemical reactions, gas dynamics, etc. In general, it is difficult to solve singular boundary value problems because of a singularity at the boundary point x= 0. Many techniques have been proposed in the literature to handle this type of singular problems.Sabir et al. [1] presented hybrid combination of genetic algorithm (GA) and interior point method (IPA) to solve Lane-Emden problems. Roul and Biswal [2] applied homotopy analysis method to solve these singular BVPs. Khuri and Sayfy [3] suggested a modified decomposition method in combination with the cubic B-spline collocation technique for the numerical solution of these equations. Roul et al. [4] developed a high order compact finite difference method for solving singular BVPs arising in various physical models. Roul et al. [5] developed a sixth-order numerical method based on sextic B-spline basis functions to solve strongly nonlinear BVPs governing electro hydrodynamic flow in a circular cylindrical conduit. Khaleghi et al. [6] implemented a new reproducing kernel method with Chebyshev polynomials for solving a class of singular BVPs. Iqbal et al. [7] applied optimal homotopy asymptotic method for the analytic solution of singular Lane-Emden equation. Arqub et al. [8] have considered the application of continuous GA for solving second order singular BVPs. Malik et al. [9] suggested a hybrid algorithm between the (GA) and (IPA) to solve some problems of singular BVPs arising in physiology applications. Baishya [10] presented hybridizing the differential transform method (DTM) with Adomian polynomials to solve nonlinear singular initial value problems. Kharrat et al. presented a hybrid genetic programming with homotopy perturbation method for solving nonlinear heat transfer equations [11].

In the previous works, the approximate solution can be expressed as many forms such as, an Adomian polynomial [10], Bernoulli polynomials [12], and a linear combination of log sigmoid function [9] has the following form:

$$\varphi(x) = \frac{1}{1+e^{-x}} \tag{1}$$

However, in our work, we will express the approximate solution as a polynomial of n degree.To apply genetic algorithm, we design windows form application in Visual C# (Microsoft Visual Studio Professional 2015, Version 14.0.25420.01).

The rest of the paper is ordered as follows. In Section 2, a brief description of the basic principles of genetic algorithm. In Section 3, we construct the proposed methodology to obtain numerical solution of singular BVPs. In Section 4, we provide some nonlinear singular BVPs arising in physiology applications and presented numerical results. Finally, Section 5 is presented our conclusions and remarks.

GENETIC ALGORITHM (GA)

Genetic Algorithm (GA) is a stochastic global search method that belongs to the large part of evolutionary algorithms[13]. GA is based on the ideas of natural selection and genetics. This is intelligent exploitation of random search, GA simulates the process of natural selection "survival of the fittest" which means those species who can adapt to changes in their environment are able to survive, reproduce and go to next generation, each generation consists of a population of individuals. Each individual represents a point in search space and potential solution. Each individual is represented as a string of character/ integer/ float/ bits. This string is analogous to the Chromosome, a possible solution to the optimization problem is coded to each chromosome, real valued coding will be used. The fitness function is used to measure the performance of a chromosome. Crossover is used where two chromosomes (parents) are fused to generate two new chromosomes (offspring), mutation generates random changes in the population. Algorithmically, GA comprises the following steps [13]:

- **Step 1**: Represent the problem variable domain as a chromosome of a fixed length.
- **Step 2**: Define a fitness function.
- **Step 3**: Randomly generate an initial population.
- **Step 4**: Calculate the fitness of each chromosome.
- **Step 5**: Select a pair of chromosomes for mating from the current population. Parent are selected with a probability related to their fitness.
- **Step 6**: Create pair of offspring chromosomes by applying genetic operators (crossover & mutation).
- **Step 7**: Place the created offspring chromosomes in the new population.
- **Step 8**: Repeat Step 5 until the size of the new chromosome population becomes equal to the size of the initial population.
- **Step 9**: Replace the parent chromosome population with the new offspring.
- **Step 10**: Go to Step 4, and repeat the process until the termination criterion is satisfied. Fig. 1 shows the flowchart of the genetic algorithm.

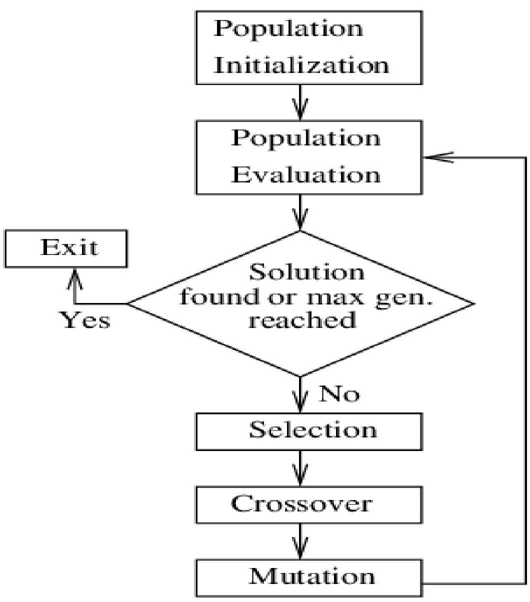

Fig.1: Genetic algorithm flowchart.

The pseudo code of the genetic algorithm is as follows [14]:

```
GA()
initialize population
find fitness of population
while (termination criteria is reached) do
    parent selection
    crossover with probability pc
    mutation with probability pm
    decode and fitness calculation
    survivor selection
    find best
return best
```

The GA method differs from other methods and the most influence differences are[14]:

- GA does not need any modification or transformation to the differential equation as in the differential transform method (DTM).
- GA is a global search algorithm that does not depend on building its initial approximation from the initial condition, as in the homotopy perturbation method.
- GA searches a population of points, not a single point.
- GA provides satisfactory solutions where it searches to find optimal solutions.
- GA can be used for solving singular BVPs whatever the type of the singular value.
- GA uses probabilistic rules, not deterministic ones.

METHODOLOGY

In this section, we describe the proposed GA based on polynomial of n degree for solving nonlinear singular BVPs.

We consider the following class of singular two-point boundary value problem [2]:

$$\begin{cases} y``(x) + \frac{m}{x} y`(x) = f(x,y) \quad ,0 < x \leq 1 \\ y`(0) = 0 \\ \mu y(1) + \sigma y`(1) = B \end{cases} \tag{2}$$

Here, $\mu > 0, \sigma \geq 0$ and B is a finite constant. The following conditions have been imposed on the function $f(x,y)$ [15]:

- $f(x,y)$is continuous function for all $(x,y) \in ([0,1] \times R)$
- $\frac{\partial f(x,y)}{\partial y}$ exists and is continuous for all $(x,y) \in ([0,1] \times R)$
- $\frac{\partial f(x,y)}{\partial y} \geq 0$

The existence and uniqueness of the solution to the problem (2) with boundary conditions have been established in [15].

We will find exact or approximate solution to this problem as a polynomial of n degree, such as:

$$y(x) = \sum_{i=0}^{n} a_i x^i \tag{3}$$

Then, the problem may be seen as optimizing the coefficients a0, a1 , … , an that minimize the fitness function.

This optimization problem can be solved using GA.The fitness function corresponding to problem maybe written in the following form [15]:

$$\varepsilon_1 = \frac{1}{N}\sum_{i=1}^{N}(y``(x_i) + \frac{m}{x_i}y`(x_i) - f(x_i, y(x_i)))^2$$

$$\varepsilon_2 = \frac{1}{2}[(y(0))^2 + (y`(1))^2]$$

$$Minimization \leftarrow FitnessFunction\varepsilon = \varepsilon_1 + \varepsilon_2$$

Where N is the number of discretization of the interval [0, 1], $\varepsilon_1$is the mean square error of differential equation (2), ε_2 is the mean square error of boundary conditions of (2), ε is the fitness function represents the mean square error of (2), subject to the availability of the parameters, such that $\varepsilon \to 0$, in case of both $\{\varepsilon_1, \varepsilon_2\} \to 0$, then the approximate results closer to the exact solution [15]. To find the best solution for the previous optimization problem, we will use the GA for obtaining the coefficients that optimize the fitness function.

NUMERICAL EXAMPLES

In this section, we will explain the feasibility of applying the proposed GA and its effectiveness through some singular boundary value problems.

1. **Example (1):**

Consider the nonlinear singular BVP arising in heat conduction model of the human head [16]:

$$\begin{cases} y``(x) + \frac{2}{x}y`(x) = -e^{-y(x)} \ , \quad 0 < x \le 1 \\ y`(0) = 0 \quad , \qquad 0.1\, y(1) + y`(1) = 0 \end{cases} \tag{4}$$

To implement the GA, we design a windows form application in visual C # as in Fig. 2:

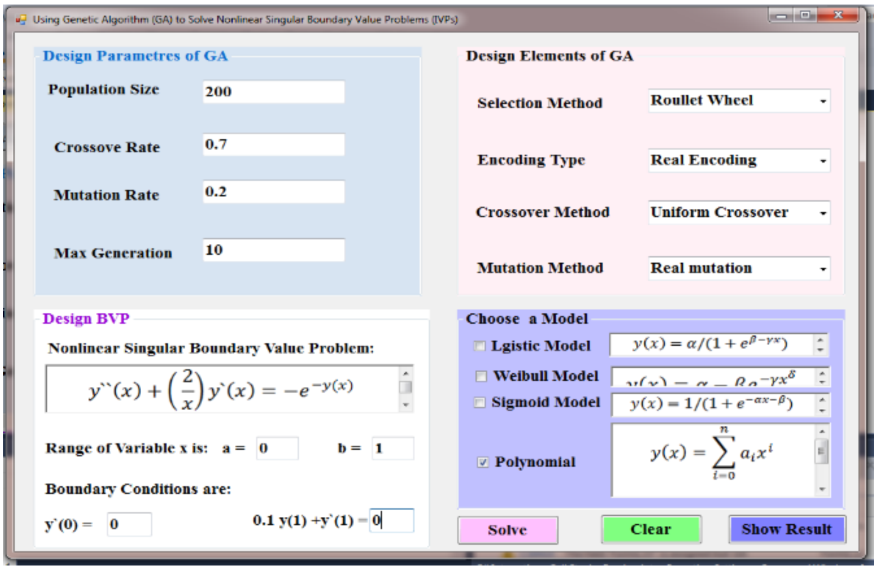

Fig.2: Windows application C# for example 1.

By observing the application in Fig 2, the best values of the GA parameters and their components are shown in the Table 1:

Table.1: Elements and parameters of the GA for Example 1.

GA parameters		GA elements	
Population size	200	Selection method	Roulette wheel
Crossover rate	0.25	Encoding	Real values
Mutation rate	0.003	Crossover method	Uniform
No. Generation	50	Mutation Method	Real mutation

The output of the previous C# application provided us with optimal values for the parameters of GA, and by substituting these values we obtained the approximate solution as follows:

$$y(x) = 3.73955963289556 - 0.384071342051807 \times 10^{-2} x^2 + 0.116426835332322 \times 10^{-6} x^4 - 0.402748233875346 \times 10^{-5} x^6$$

In Table 2, we present a comparison of the absolute errors between our presented method and the method given in [16].

Table.2: Comparison of the absolute error for Example 1.

x	Presented GA	Taylor series Method in [16]
0.0	0	0
0.1	0.6640218228 E-3	0.6833052562
0.2	0.6666001467 E-3	0.6835628813
0.3	0.6702305722 E-3	0.6839925764
0.4	0.6738933595 E-3	0.6845948216
0.5	0.6761649576 E-3	0.6853702912
0.6	0.6752263913 E-3	0.6863198539
0.7	0.6688545625 E-3	0.6874445750
0.8	0.6544056189 E-3	0.6887457192
0.9	0.6288401722 E-3	0.6902247519
1.0	0.5887303713 E-3	0.6918833436

2. **Example (2):**

Consider the nonlinear singular BVP arising in the study of oxygen diffusion in a spherical cell [16]:

$$\begin{cases} y``(x) + \frac{2}{x} y`(x) = \frac{0.76129\, y(x)}{y(x) + 0.03119}, \quad 0 < x \leq 1 \\ y`(0) = 0 \\ 5\, y(1) + y`(1) = 5 \end{cases} \tag{5}$$

Fig. 3. shows the application C# for design the GA:

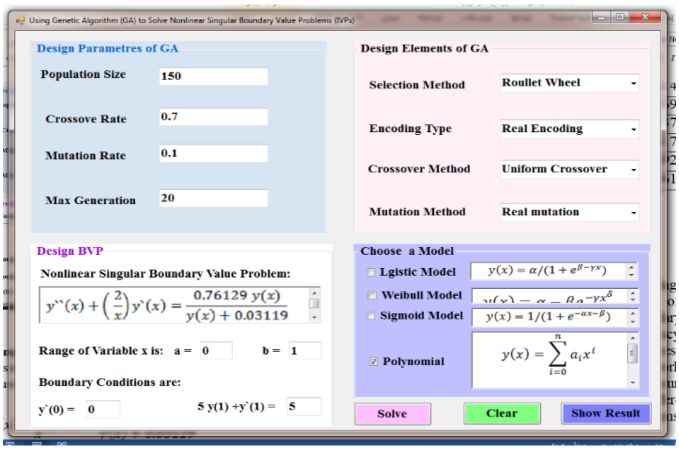

Fig. 3. Windows application C# for example 2.

The output of the previous C# application provided us by the optimal values for the parameters of the GA, the approximate solution obtained by the GA is shown as:

$$y(x) = -1.23616526233311 \times 10^{-13} - 2.85947149866421 \times 10^{-13}x^2 + 2.62545244536 \times 10^{-13}x^4 - 2.04125752067762 \times 10^{-11}x^6 - 8.4891999315968 \times 10^{-13}x^8$$

Table 3 shows a comparison between the results obtained by our proposed GA, the method used in [16] and the method given in [17].

Table.3: Comparison of the absolute error for Example 2.

x	Presented GA	Taylor series Method in [16]	Homotopy analysis Method in [17]
0.0	0	0	0
0.1	1.274492551 E -12	1. E -10	3.2902951 E-03
0.2	5.78234854 E -13	7.4 E -09	3.1682656 E-03
0.3	9.412722876 E -12	8.36 E -08	2.9644033 E-03
0.4	3.329029302 E -11	4.719 E -07	2.6780358 E-03
0.5	8.236457320 E -11	1.8148 E -06	2.3083081 E-03
0.6	1.677141866 E -10	5.4692 E-06	1.8542621 E-03
0.7	2.997917956 E -10	1.39375 E-05	1.3149118 E-03
0.8	4.864519062 E -10	3.14178 E-05	6.893301 E-04
0.9	7.305114814 E -10	6.44925 E-05	2.32886 E-05
1.0	1.026789061 E -9	1.229619 E -04	8.235730 E-04

3. Example (3):

Consider another form of the nonlinear singular BVP arising in the study of oxygen diffusion [18]:

$$\begin{cases} y``(x) + \dfrac{3}{x} y`(x) = \dfrac{0.76129\, y(x)}{y(x) + 0.03119}, & 0 < x \leq 1 \\ y`(0) = 0 \ , \ 5\, y(1) + y`(1) = 5 \end{cases} \qquad (6)$$

The approximate solution generated by the GA is:

$$y(x) = -1.23616526233311 \times 10^{-13} - 2.85947149866421 \times 10^{-13}x^2 + 2.62545244536 \times 10^{-13}x^4 - 2.04125752067762 \times 10^{-15}x^6 - 8.4891999315968 \times 10^{-15}x^8$$

Table 4 illustrates a comparison between the results obtained by our presented GA and the method given in [18].

Table.4: Comparison of the absolute error for Example 3.

x	Presented GA	Adomian decomposition Method in [18]
0.0	0	0
0.1	7.98388439 E-13	1.378 E-07
0.2	2.35431397 E-13	1.212 E-07
0.3	3.266727030 E-12	9.62 E-08
0.4	1.355464649 E-11	6.49 E-08
0.5	3.601478930 E-11	3.10 E-08
0.6	7.757444855 E-11	4.4 E-09
0.7	1.467024395 E-10	4.06 E-08
0.8	2.534091801 E-10	7.77 E-08
0.9	4.092456974 E-10	1.175 E-07
1.0	6.273010235 E-10	1.604 E-07

CONCLUSION

In this article, we proposed expanding the application of genetic algorithm as one of the meta-heuristic algorithms to solve nonlinear singular boundary value problems. The accuracy and effectiveness of the proposed algorithm are illustrated by solving some nonlinear singular BVPs arising in physiology applications. In addition, the GA can find the approximate numerical solution for the singular BVPs where the traditional methods cannot be formulated the exact solution.

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