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Hybrid Evolutionary Algorithm with Homotopy Perturbation Method for Solving Nonlinear Heat Transfer Equations

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Abstract: *In this paper, we present a hybrid method for solving nonlinear heat transfer equations, by using genetic programming (GP) with homotopy perturbation method (HPM). The main advantage of hybrid method is that the obtained solution leads to a small error solution at all points in the solution interval. While the resulting solution obtained by applying the traditional methods leads to error increase as we move away from the beginning of the solution interval. The solution that can be obtained by hybrid technique depends on generate initial approximation from GP then applied HPM to find other terms of the solution series. The results of the proposed technique are compared with standard HPM, which witnesses the effectiveness and viability of the suggested hybrid algorithm.*

Keywords **-***Boundary value problem(BVP), Evolutionary Algorithm (EA), Genetic Programming (GP), Homotopy Perturbation Method (HPM), Initial Value Problem (IVP), Nonlinear Heat Transfer Equations.*

I. INTRODUCTION

Most scientific problems and phenomena such as heat transfer occur nonlinearly. Several analytical and numerical techniques have suggested by researchers for solving nonlinear heat transfer equations as quoted under references [1–19].

The heat transfer equations have major practical importance in cooling of electronic components, cooling of heated stirred vessels and heated parts of the space vehicles etc. [1].This work is mainly concerned to present a hybrid method for the study of two nonlinear heat transfer equations. First equation refers to a well known boundary value temperature distribution equation. While second equation is regarded as an initial value cooling equation.

Recently, Ganji et al. [2] tailored nonlinear heat transfer equations by homotopy perturbation method (HPM) and variational iteration method (VIM). A. Atangana in [3] adopted a new iterative analytical technique along with the stability, convergence and the uniqueness analysis of adopted technique for dealing with nonlinear fractional partial differential equations arising in biological population dynamics system. Abbasbandy [1] encountered nonlinear heat transfer equations by using homotopy analysis method (HAM).[4] employed semi-analytical methods i.e. homotopy perturbation method (HPM) and finite difference method (FDM) which enhances heat transfer and validates semi-analytical methods for study of heat transfer rates while Saeed et al. [5] obtained the solutions of nonlinear heat transfer equations by using Haar Wavelet- Quasilinearization Technique.

Because of limitations in accuracy and efficiency of these classical analytical and numerical methods, the evolutionary algorithms have demonstrated their importance, investigated thoroughly, applied successfully to solve nonlinear problems in engineering, and applied science domain [6-7]. For instance, Arqub et al. [8-9] considered continuous genetic algorithm based technique for dealing with boundary value problem, Troesch's and Bratu's Problems. Malik et al. [10-11] taking into account an evolutionary computing scheme of hybrid genetic algorithm (HGA) for solving biochemical reaction and singular boundary value problems arising in Physiology. Kadri et al. [12] solved nonlinear heat conduction problems by using genetic algorithm (GA). Arqub et al. in [13] investigated the efficiency, accuracy and convergence analysis of the continuous genetic algorithm by solving a class of nonlinear systems of second order boundary value problems. Ullah A, Malik SA and Alimgeer in [14] proposed a hybrid heuristic evolutionary scheme for nonlinear heat transfer equations. In [15] Eman A. H and Yaseen M. A presented an accelerated genetic algorithm to solve partial differential equations. Tom Seaton1, Gavin Brown2, and Julian F. Miller [16] solved system of ordinary differential equations using Graph-Based Genetic Programming. Koza briefly addressed learning solutions to ODEs in his seminal work on problems for Tree Genetic Programming [17].

We are also aware of a novel hybrid GP approach to solving differential equations implemented by Kirstukaset. al [18]. Ahmed Entesar, Omar Saber and Waleed Al-Hayani in [19] presented Hybridization of Genetic Algorithm with Homotopy Analysis Method for Solving Fractional Partial Differential Equations.

We designed windows form application in Visual c#to apply genetic programming, and used Maple 12.0 software for HPM.

Remainder of the paper is orderly as follows: in section 2 description of the evolutionary algorithms. In section 3, we have the genetic programming. In section 4, we have the basic ideas of the homotopy perturbation method. In section 5, we have the proposed hybrid method (GP-HPM). In section 6, we have examples (1 and 2) of the nonlinear heat transfer equations. Finally, section 7 concludes the paper.

II. EVOLUTIONARY ALGORITHMS

Evolutionary algorithms (EAs) are the population-based metaheuristic optimization algorithms [20]. Candidate solutions to the optimization problem are defined as individuals in a population, and evolution of the population leads to finding better solutions. The fitness of individuals to the environment is estimated and some mechanisms inspired by biological evolution are applied to evolution of the population. Genetic algorithm (GA), Evolution strategy (ES), Genetic programming (GP), and Evolutionary programming (EP) are very popular Evolutionary algorithms.

Evolutionary algorithms are successively applied to wide optimization problems in the engineering, marketing, operations research, and social science, such as include scheduling, genetics, material selection, structural design and so on. Apart from mathematical optimization problems, evolutionary algorithms have also been used as an experimental framework within biological evolution and natural selection in the field of artificial life [18].Evolutionary algorithms are easy to implement and often provide adequate solutions. An origin of these algorithms is found in the Darwin principles of natural selection (Darwin, 1859). In accordance with these principles, only the fittest individuals can survive in the struggle for existence and reproduce their good characteristics into next generation. As illustrated in Fig. 1, evolutionary algorithms operate with the population of solutions. At first, the solution needs to be defined within an evolutionary algorithm. Usually, this definition cannot be described in the original problem context directly. In contrast, the solution is defined by data structures that describe the original problem context indirectly and thus, determine the search space within an evolutionary search (optimization process) [18]. Before an evolutionary process actually starts, the initial population needs to be generated. The initial population is generated most often randomly. A basis of an evolutionary algorithm represents an evolutionary search in which the selected solutions undergo an operation of reproduction, i.e., a crossover and a mutation. As a result, new candidate solutions (offsprings) are produced that compete, according to their fitness, with old ones for a place in the next generation. The fitness is evaluated by an evaluation function that defines requirements of the optimization. As the population evolves solutions becomes fitter and fitter. Finally, the evolutionary search can be iterated until a solution with sufficient quality (fitness) is found or the predefined number of generations is reached [20].

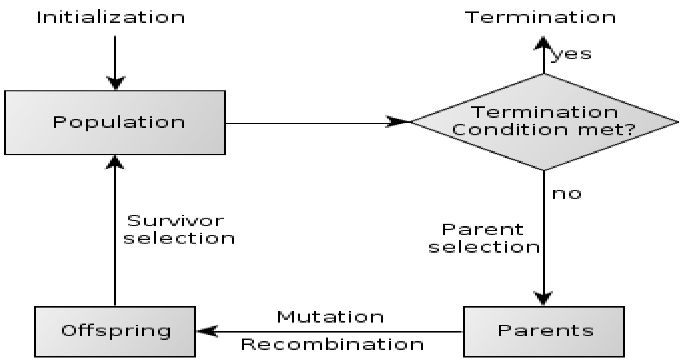

Fig.1: Scheme of Evolutionary Algorithm

III. GENETIC PROGRAMMING (GP)

Genetic programming (GP), suggested by Koza (1992b) emerged in the early 1990s. GP explicitly performs the optimization of programs. GP is a systematic method for getting computers to automatically solve a problem starting from a high-level statement of what needs to be done. Genetic programming is a domain-independent method that genetically breeds a population of computer programs to solve a problem [21]. Specifically, genetic programming iteratively transforms a population of computer programs into a new generation of programs by applying analogs of naturally occurring genetic operations [21].

Genetic programming is an extension of the genetic algorithm (Holland 1975) in which the structures in the population are not fixed-length character strings that encode candidate solutions to a problem, but programs that, when executed, are the candidate solutions to the problem [22].

3.1. Representation:

Programs are expressed in genetic programming as syntax trees rather than as lines of code. For example, the simple expression max(x*x,x+3*y)is represented as shown in Fig 2. The tree includes nodes (which we will also call point) and links. The nodes indicate the instructions to execute. The links indicate the arguments for each instruction. In the following the internal nodes in a tree will be called functions, while the tree's leaves will be called terminals [21].

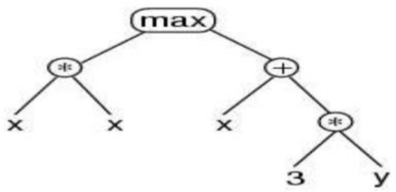

Fig.2: GP syntax tree representing max(x+x, x+3*y)

Genetic programming trees and their corresponding expressions can equivalently be represented in prefix notation (e.g., as Lisp S-expressions). In prefix notation, functions always precede their arguments. For example, max(x*x,x+3*y) becomes (max (* x x)(+ x (* 3 y))). In this notation, it is easy to see the correspondence between expressions and their syntax trees. In this paper, we used trees representation and their corresponding infix-notation expressions [21].

3.2. Initializing the Population:

Like in other evolutionary algorithms, in GP the individuals in the initial population are typically randomly generated. There are a number of different approaches to generating this random initial population. In this paper, we use Grow method.

In Grow method, the initial individuals are generated subject to a pre-established maximum depth, the grow method allows for the creation of trees of varying size and shape. The nodes are selected from the whole primitive set (functions and terminals) until the depth limit is reached. Fig. 3 illustrates this process for the construction of a tree with depth limit 2.

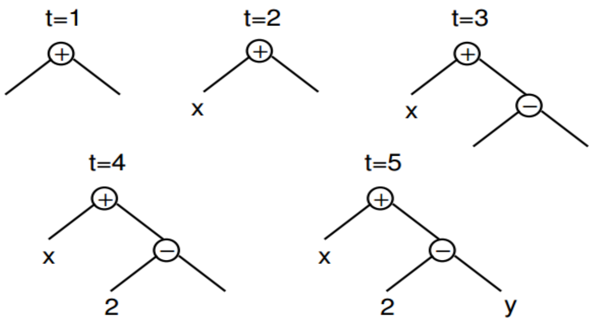

Fig.3: Creation a five node tree using grow initialization method with maximum depth of 2 (t=time)

3.3. Crossover:

The most commonly used form of crossover is subtree crossover. Given two parents, subtree crossover randomly (and independently) selects a crossover point (a node) in each parent tree. Then, it creates the offspring by replacing the subtree rooted at the crossover point in a copy of the first parent with a copy of the subtree rooted at the crossover point in the second parent [23].

3.4. Mutation:

The most commonly used form of mutation in GP (which we will call subtree mutation) randomly selects a mutation point in a tree and substitutes the subtree rooted there with a randomly generated subtree. When subtree mutation is applied, this involves the modification of exactly one sub tree. Point mutation, on the other hand, is typically applied on a per-node basis. That is, each node is considered in turn and, with a certain probability, it is altered as explained above. This allows multiple nodes to be mutated independently in one application of point mutation [23].

3.5. Preparatory Steps to run GP:

The five major preparatory steps for the basic version of genetic programming require the human user to specify [21],

1. The set of terminals for each branch of the to-be evolved program.

2. The set of primitive functions for each branch of the to-be-evolved program.

3. The fitness measure (for explicitly or implicitly measuring the fitness of individuals in the population).

4. Certain parameters for controlling the run.

5. The termination criterion and method for designating the result of the run.

Table 1 shows a sample of some of the functions one sees in the GP literature[21].

Table 1: Examples of primitives in GP function and terminal sets

Primitive Set			
Function Set		**Terminal Set**	
Kind of Function	**Example (s)**	**Kind of Terminal**	**Example (s)**
Arithmetic	+, *, -, %, ….	Variables	x, y, …
Mathematical	sin, cos, exp, log …	Constant values	3, 0.45
Boolean	And, Or, Not…	0-arity functions	rand(), go-left…
Conditional	IF-Then-ELSE		
Loop	FOR-REPEAT …		

3.6. Initial steps of GP:

Genetic programming iteratively transforms a population of computer programs into a new generation of the population by applying analogs of naturally occurring genetic operations. These operations are applied to individual(s) selected from the population. The individuals are probabilistically selected to participate in the genetic operations based on their fitness. The iterative transformation of the population is executed inside the main generational loop of the run of genetic programming [22]. Algorithmically, GP comprises the steps shown as follow [21]:

a) Randomly create an initial population of programs (from the available primitives)
b) Repeat
c) Execute each program and ascertain its fitness.
d) Select one or two program (s) from the population with a probability based on fitness to participate in genetic operations.
e) Create new individual program (s) by applying genetic operations with specified probabilities.
f) Until an acceptable solution is found or some condition is met.
g) Return the best_ so_ far individual.

IV. BASIC IDEAS OF THE HOMOTOPY PERTURBATION METHOD

The Homotopy Perturbation method was proposed first by J.H. He [24] for solving linear and non linear boundary value problems and initial value problems. To illustrate the basic ideas of this method, we consider the following general non linear differential equation:

$$A(u) - f(r) = 0 \quad , \quad r \in \Omega \tag{1}$$

With the following boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \quad , \quad r \in \Gamma \tag{2}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytical function and Γis the boundary of the domain Ω. The operator A can be decomposed into a linear part and a nonlinear one, designated as L and N respectively. Hence Eq. (1) can be written as the following form:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

Using HPM technique, we construct a homotopy $v(r,p): \Omega \text{ X } [0,1] \to R$, which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{4}$$

Where $p\epsilon[0,1]$ is an embedding parameter and u_0 is an initial approximation of Eq. (1) which satisfies the boundary conditions. Obviously, from Eq. (4) we have [24]:

$$H(v,0) = L(v) - L(u_0) = 0$$

$$H(v,1) = A(v) - f(r) = 0$$

By changing the value of p from zero to unity, v(r, p) changes from $u_0(r)$ to u(r), in topology this called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopic. Due to the fact that $p\epsilon[0,1]$can be considered as a small parameter, hence we consider the solution of Eq. (4) as a power series in p as the following:

$$v = v_0 + pv_1 + p^2v_2 + \cdots \tag{5}$$

Setting p = 1 results in the approximate solution for Eq. (1) [24]

$$u = \lim_{p\to1} v = v_0 + v_1 + v_2 + \cdots \tag{6}$$

V. PROPOSED HYBRID METHOD GP-HPM

In this work, the GP algorithm is exploited to find the initial approximation for the nonlinear heat transfer equation. The choice of this technique is motivated by the following factors. First, GPcan provide solutions for highly complex search spaces also. Second, GP can be performed well approximating solutions to all types of problems [12].After that, we select the best individual from GP and add it to other terms of solution series in HPM method.

The advantage of this proposed hybrid method is we can control the solution and obtain a series close to the exact solution. Table 2 shows the steps for hybrid GP-HPM.

Table 2: Steps for hybridization of GP with HPM

Step 1 (population Initialization)	Random population of N individuals
Step 2 (Evaluation)	Evaluated each individual using fitness function
Step 3 (Stopping Conditions)	The algorithm keeps executing until reach to the maximum number of generations
Step 4 (Selection &Crossover)	Based on fitness value, the individuals from current population are chosen as parents for new generation. These parents then produce further offspring as a result of crossover operation which became parents for next generation.
Step 5 (Mutation)	It randomly changes the offspring resulted from crossover to find a good solution.

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Step 6 (Improvement)	The optimal individual found by GP fed to HPM as an initial approximation which tends to optimize the results further.

VI. NONLINEAR HEAT TRANSFER EQUATIONS

6.1. Example (1):

The initial value cooling equation of a lumped system by combined convection and radiation whose governing equation is given by [14]:

$$x'(t) + x(t) + 0.6\big(x(t)\big)^4 = 0 \quad , \quad t \in [0,1] \tag{7}$$

With the initial condition:

$$x(0) = 1$$

6.1.1. By using GP algorithm:

The given nonlinear IVP of Eq. (7)is converted into an error minimization problem as follows [12]:

$$\varepsilon_1 = \frac{1}{N+1}\sum_{i=0}^{N}\left(x'(t_i) + x(t_i) + 0.6\big(x(t_i)\big)^4\right)^2 \tag{8}$$

$$\varepsilon_2 = (x(0) - 1)^2 \tag{9}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{10}$$

Where N is the total number of steps taken in the solution range [0, 1]. $\varepsilon_1$is the mean of the sum of square of the give Eq. (8), ε_2 is the mean of the sum of square error due to initial condition of Eq. (9). εisthe fitness function represents the sum of the mean square error of the given ordinary differential equation and its initial condition of Eq. (10). The initial approximation was obtained from GP algorithm by using the following parameters as shown in Table 3:

Table 3: Parameters for Exp(1) GP run

Objective	Find the best initial approximation
Representation	Syntax tree
Function Set	+,*,-,%,sin, cos, tan, exp, log, ln, sqrt,^
Terminal Set	t , and constants chosen randomly between -10 and +10
Fitness	Sum of absolute errors for t Є {0.1, 0.2, … 0.9, 1.0}
Initial Population	Grow method
Control Parameters	Population size 100, 25% subtree crossover, 1% subtree mutation, max depth of tree 5, No. of generations 50.

We obtained the following initial approximation:

$$x_0 = \ln(\sin(1)) \tag{11}$$

6.1.2. By using the HPM:

The homotopy of Eq. (7) can be written as follows:

$$H(x,p) = (1-p)(x' - u_0') + p(x' + x + 0.6x^4) = 0$$

Or

$$x' - u_0' + p(u_0' + x + 0.6x^4) = 0 \tag{12}$$

Where $p\epsilon[0,1]$is an embedding parameter.
According to the HPM the solution of Eq. (12) can be written as a power series in p

$$x = \sum_{i=0}^{\infty} p^i x_i \tag{13}$$

Substituting Eq. (13) into Eq. (12), we obtain:

$$\sum_{i=0}^{\infty} p^i x_i' - u_0' + p\left(u_0' + \sum_{i=0}^{\infty} p^i x_i + 0.6(\sum_{i=0}^{\infty} p^i x_i)^4\right) = 0 \tag{14}$$

And comparing coefficients of terms with identical powers of pin the result system, leads to:

$$p^0: x_0' - u_0' = 0 \tag{15}$$

$$p^1: x_1' + u_0' + x_0 + 0.6\, x_0^4 = 0 \tag{16}$$

$$p^2: x_2' + x + 0.6(4x_0^3 x_1) = 0 \tag{17}$$

From Eq. (15) we obtained:

$$u_0 = x_0 = 1$$

From Eq.(16):

$$x_1(t) = -1.600000000\, t$$

From Eq.(17):

$$x_2(t) = 2.720000000\, t^2$$

Then we get the approximate solution by HPM as follow:

$$x(t) = x_0(t) + x_1(t) + x_2(t) = 1 - 1.600000000\, t + 2.720000000\, t^2 \tag{18}$$

6.1.3. By using Hybrid method GP-HPM:

From (11):

$$x_0 = \ln(\sin(1)) \approx -0.1726037462690916785$$

From Eq.(16):

$$x_1 = 0.1720712064\, t$$

From Eq.(17):

$$x_2 = -0.08497380960\, t^2$$

Then we get the approximate solution by hybrid GP-HPM as follow:

$$x(t) = x_0(t) + x_1(t) + x_2(t) = -0.1726037462690916785 + 0.1720712064\, t - 0.08497380960\, t^2 \tag{19}$$

In the following Table4 we give the differences among the approximate solution of(19) by taking three terms of Hybrid and three terms of HPM from (18) of the solution series.

Table 4: The comparison of the errors by HPM and proposed GP-HPM for Example 1

t	Error of HPM (n=3) $x_0(t) + x_1(t) + x_2(t)$	Error of Hybrid GP-HPM (n=3) $x_0(t) + x_1(t) + x_2(t)$
0.0	0.0000E+00	3.8300000 E -11
0.1	1.50535E-01	8.12324697E-04
0.2	5.09084E-01	3.26563851E-03
0.3	10.0208E-01	7.37885081E-03
0.4	16.1111E-01	1.31650482E-02
0.5	33.3062E-01	2.06331375E-02
0.6	47.1890E-01	2.97890751E-02
0.7	47.1890E-01	4.06367657E-02
0.8	69.4501E-01	5.31787014E-02
0.9	108.583E-01	6.74164003E-02
1.0	180.798E-01	8.33506889E-02

6.2. Example (2):

The boundary value temperature distribution equation in a lumped system of combined convection-radiation that can be described by the following mathematical model [14]:

$$x''(t) - 0.6\big(x(t)\big)^4 = 0 \quad , \quad t \in [0,1] \tag{20}$$

With the boundary conditions:

$$x'(0) = 0 \quad \text{and} \quad x(1) = 1$$

6.2.1. By using GP:

The given nonlinear BVP of Eq. (20)is converted into an error minimization problem as follows [12]:

$$\varepsilon_1 = \frac{1}{N+1}\sum_{i=0}^{N}(x''(t_i) - 0.6x^4(t_i))^2 \tag{21}$$

$$\varepsilon_2 = \frac{1}{2}((x'(0) - 0)^2 + (x(1) - 1)^2) \tag{22}$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{23}$$

Where N is the total number of steps taken in the solution range [0, 1]. $\varepsilon_1$is the mean of the sum of square of the give Eq (21), ε_2 is the mean of the sum of square error due to boundary conditions of Eq(22). εisthe fitness function represents the sum of the mean square error of the given ordinary differential equation and its boundary conditions

6.2.2. the initial approximation was obtained from GP algorithm by using the following parameters as shown in Table 5:

Table 5: Parameters for Exp. (2) GP run

Objective	Find the best initial approximation
Representation	Syntax tree
Function Set	+,*,-,%,sin, cos, tan, exp, log, ln, sqrt,^
Terminal Set	t , and constants chosen randomly between -10 and +10
Fitness	Sum of absolute errors for t Є {0.1, 0.2, … 0.9, 1.0}
Initial Population	Grow method
Control Parameters	Population size 1000, 50% subtree crossover, 3% subtree mutation, max depth of tree 5, No. of generations 100.

We obtained the following initial approximation:

$$x_0(t) = e^{-10} \tag{24}$$

6.2.3. By using HPM:

The homotopy of Eq. (20) can be written as follows:

$$H(x,p) = (1-p)(x'' - u_0'') + p(x'' - 0.6x^4) = 0$$

Or:

$$x'' - u_0'' + p(u_0'' - 0.6x^4) = 0 \tag{25}$$

Substituting Eq. (13) into Eq. (25) we obtain:

$$\sum_{i=0}^{\infty} p^i\, x_i'' - u_0'' + p\left(u_0'' - 0.6(\sum_{i=0}^{\infty} p^i\, x_i'')^4\right) = 0$$

And comparing coefficients of terms with identical powers of p in the result system, leads to:

$$p^0 : x_0'' - u_0'' = 0 \tag{26}$$

$$p^1 : x_1'' + u_0'' - 0.6\, x_0^4 = 0 \tag{27}$$

$$p^2 : x_2'' - 0.6(4\, x_0^3\, x_1) = 0 \tag{28}$$

From Eq.(26):

$$x_0 = x(0) + x'(0).t = \alpha$$

Where $x(0) = \alpha$

By using $x(1) = 1$ leads to $\alpha = 1$

Then

$$x_0(t) = 1$$

From Eq(27):

$$x_1(t) = \frac{3}{10}t^2$$

From Eq(28):

$$x_2(t) = \frac{3}{50}t^4$$

Then we get the approximate solution by HPM as follow:

$$\mathbf{x}(t) = x_0(t) + x_1(t) + x_2(t) = 1 + \frac{3}{10}t^2 + \frac{3}{50}t^4 \tag{29}$$

6.2.4. By using Hybrid method GP-HPM:

Substituting the initial approximation from (24) into Eq. (27) we obtain:

$$x_1(t) = \frac{3}{10}e^{-40}t^2$$

Then we get the approximate solution by hybrid GP-HPM as follow:

$$x(t) = x_0(t) + x_1(t) = e^{-10} + \frac{3}{10}e^{-40}t^2 \quad (30)$$

Table 6indicates the comparison between the errors resulting by HPM and proposed hybrid GP-HPM.

Table 6: The comparison of the errors by HPM and proposed GP-HPM for Example 2

t	Error of HPM (n=3) $x_0(t)+x_1(t)+x_2(t)$	Error of Hybrid GP-HPM (n=2) $x_0(t)+x_1(t)$
0.0	0.00E+00	6.16897 E -028
0.1	4.70 E -05	6.16901 E -028
0.2	7.61 E -04	6.16910 E -028
0.3	3.94 E -03	6.16923 E -028
0.4	1.28 E -02	6.16943 E -028
0.5	3.25 E -02	6.16969 E -028
0.6	7.07 E -02	6.17001 E -028
0.7	1.39 E -01	6.17038 E -028
0.8	2.53 E -01	6.17080 E -028
0.9	4.39 E -01	6.17128 E -028
1.0	7.33 E -01	6.17183 E -028

VII. CONCLUSION

In this paper, the GP technique has been used for obtaining the initial approximation, which used to hybrid GP-HPM. The results show that the hybrid method as great accuracy and its leads to an analytical solution with a small error in all points of the solution range, while the traditional method leads to solution with errors that increase as we move away from the beginning of the solution range. Where GP exploited as a global search optimization method for generate an initial approximation, but the initial approximation from HPM was constructed depends on the initial conditions. We noted that the proposed GP-HPM is successful in giving accurate solution of nonlinear heat transfer equations. It is established that the proposed technique is a good and trusty alternative approach for researchers for solving nonlinear problems in engineering and other applied sciences domain.

REFERENCES

[1] M. Danish , S. Kumar and Sh. Kumar ,Exact Solutions of Three Nonlinear Heat Transfer Problems,*Engineering Letters*, 19:3, (2011)EL_19_3_15.
[2] D.D.Ganji and A.Sadighi,Application of homotopy-perturbation and variational iteration methods tononlinear heat transfer and porous media equations,*Journal of Computational and Applied Mathematics* 207,(2007)24–34.
[3] A.Atangana,Convergence and stability analysis of a novel iteration method for fractional biological population equation, *Neural Comput&Applic* 25 (5), (2014) 1021–1030.
[4] M.Dehghan, Y.Rahmani, D.D.Ganji, S.Saedodin, MS.Valipour and S.Rashidi, Convection-radiationheat transfer in solar heat exchangers filled with a porous medium, Homotopy perturbation method versus numerical analysis. *Renewable Energy* 74, (2015)448–455.
[5] U. Saeed, M. Rehman, Assessment of Haar Wavelet-Quasilinearization,Technique in Heat Convection-Radiation Equations, *Applied Computational Intelligence and Soft Computing*(2014)ID454231.
[6] S. A. Malik, A. Ullah, I.M. Qureshi, M. Amir, Numerical Solution to Duffing Equation Using HybridGenetic Algorithm Technique. *MAGNT Research Report* 3(2), (2015) 21–30.
[7] S. A. Malik , I.M. Qureshi, M. Amir, I. Haq , Numerical Solution to Generalized Burgers'-Fisher Equation Using Exp-Function Method Hybridized with Heuristic Computation,*journal PLOSE* ONE 10(3):(2015)e0121728.
[8] O. Abu Arqub, Z. Abo-Hammour, S. Momani, N. Shawagfeh,Solving Singular Two-Point BoundaryValue Problems Using Continuous Genetic Algorithm,*Abstract and Applied Analysis Article*(2012)ID 205391.
[9] Z. Abo-Hammour, O. Abu Arqub, S. Momani, N. Shawagfeh,Optimization Solution of Troesch'sand Bratu's Problems of Ordinary Type Using Novel Continuous Genetic Algorithm,*Discrete Dynamicsin Nature and Society Article*(2014)ID 401696
[10] S. A. Malik, I.M. Qureshi, M. Amir, I. Haq, Numerical Solution to Nonlinear Biochemical ReactionModel Using Hybrid Polynomial

Basis Differential Evolution Technique,*Advanced Studies in Biology* 6(3), (2014) 99–113.
[11] S. A. Malik, I.M. Qureshi, M. Zubair, I. Haq, Memetic Heuristic Computation for Solving Nonlinear Singular Boundary Value Problems Arising in Physiology,*Research Journal of Recent Sciences* 2(9), (2013) 47–55.
[12] M.B. Kadri, W.A. Khan,Application of Genetic Algorithms in Nonlinear Heat Conduction Problems,*The Scientific World Journal Article*(2014)ID 451274.
[13] O. Abu Arqub, Z. Abo-Hammour, S. Momani, Application of Continuous Genetic Algorithm for Nonlinear System of Second-Order Boundary Value Problems,*Applied Mathematics & Information Sciences* 8(1), (2014) 235–248.
[14] A. Ullah, S. Abdullah and K.S. Alimgeer, Evolutionary algorithm based heuristic scheme for nonlinear heat transfer equations,*journal PLOSE* ONE.(2018)
[15] E.A. Hussain and Y.M. Alrajhi, Solutions of partial differential equations using accelerated genetic algorithm, *International journal of mathematics and statistics studies* 2(1),(2014) 55-69.
[16] T. Seaton, G. Brown, and J.F. Miller, Analytic Solutions to Differential Equationsunder Graph-Based Genetic Programming,*Julian Francis MillerThe University of York 310 PUBLICATIONS*(2014)6,942 CITATIONS
[17] J. R. Koza, on the Programming of Computers By Meansof Natural Selection,*MIT Press*, (1992)Cambridge
[18] S.J. Kirstukas, K.M. Bryden, D.A. Ashlock, A hybrid genetic programming approach for the analytical solution of differential equations,*International Journal ofGeneral Systems* 34, (2005) 279–299
[19] A. Entesar, O. Saber, W. Al-Hayani, Hybridization of Genetic Algorithm with Homotopy Analysis Method forSolving Fractional Partial Differential Equations, *Eurasian Journal of Science & Engineering*4(2), (2018)2414-5629.
[20] E. Kita, *evolutionary algorithms*(InTech,Croatia SBN 978-953-307-171-8, 2011).
[21] R.Poli, W. B. Langdon, N. F. Mcphee and J. R. Koza,*a field guide to genetic programming*(University of Essex – UK, 2008).
[22] J. R. Koza, *on the programming of computers by means of natural selection* (themovie Koza and Rice 1992).
[23] Z.Michalewicz, *genetic programming + data structures = automatic programming*(Springer-VerlageBerline, 1998).
[24] J.H. He,A coupling method of a homotopy technique and a perturbation technique for non-linear problems,*International Journal of Non-Linear Mechanics.* 35, (2000) 37-43.
[25] S.J. Liao, *the proposed homotopy analysis technique for the solution of nonlinearproblems*, doctoral dissertation, Shanghai Jiao Tong University, 1992.