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Modeling Electrospinning Jet of a Polymeric Solution in Steady State

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ABSTRACT:*This paper discusses Modeling of steady state of electrospinning jet. Its importance comes from the possibilityof utilizingthe jet in later process by accurately control the collection point location and by providing an acceptable anticipating level of the jet radius at the collector. The proposed model was tested against experimental results and the accuracy of the model is 93%.*
The solution is relatively highly viscous and this high viscosity comes from the contributions of the polymer and not of the solvent as its viscosity relative to that of the polymer is negligible.this leads to the need to use a model that is able to account for the viscoelastic behavior of the solution
One candidate is Giesekus model as it is suitable for relatively high viscous solution in addition to its applicability for viscoelastic polymeric solutions
The paper study the influence of five parameters namely (Voltage, nozzle radius, volumetric flow, spinning distance, solution viscosity) on the radius of the jet. It concludes that the radius of the collected jet increases with the increase of volumetric flow, solution viscosity and nozzle radius, while it decreases with the increase of, Voltage, andspinning distance.

Keywords: *Electrospinning, Modeling, Experimental, viscous polymeric solutions, Giesekus model*

INTRODUCTION

The process of electrospinning is concerned with the stretching of a weakly conducting polymeric jet of a solution or melt resulted from applying an electric field on it.In comparison with conventional spinning process electrospinning replaces the conventional mechanical stretching forces applied at the end terminal of the fiber with electrical forces exerted on the hole surface of the spun jet by the interaction between an external electrical field and electrical charges distributed on the surface of the polymeric jet.This gives uniform distribution of the stretching forces resulting in rapid reduction of the jet radius.

Electrospinning process is defined by means of a set of parameters which can be divided into the following groups:

1- Rheological: these can be Viscosity of solvent, polymer and solution surface tension
2- Electrical:Solution conductivity, applied voltage and resultant electric field
3- Geometrical: these include
   - Source diameter, distance between source and collector and the shape of collector
   - Controlling these groups determine the quality of the collected fiber
   - Modeling is used as an inexpensive tool to understand the influence of the different parameters of the diameter of the fiber.

Historical brief on electrospinning technology

In the beginning of the eighteenth century Gray [1] recorded his observation on how the presence of an electrostatic field effects on water. More than a century later Larmor [2] used the principles of the electrodynamics to describe the effect of an electric charge on nonconductive liquid. In 1936, Sze et al [3], noted that a charged ring caused a water jet that the ring intersected its path to diverge while a charged sphere had an opposite effect causing the water drops that passed by it to converge, this effect was described as an electric lens acting similarly to an optical lens.

The first recorded notation about electro spinning dates back to electric field the start of the twentieth century by Cooley [4] and Morton [5].

Auxiliary electrodes were used to direct the jet and deposit it on a rotation collector. Between 1938 and 1939 Formhals [6 - 11] registered six US patents related to electro spinning using multiple spinnerets and parallel electrodes to collect aligned fibers.

An attempt to study the performance of a thin liquid jet in the presence of an electric field conducted by Zenley [12], Taylor [13, 14, and 15]

During the last decade, with the raise of interest in nanotechnology different models were developed by Reneker, Dzenis, Spivak [16,17] and Hohman [18, 19] and Feng [20] and recently researches conducted by Arumuganathar [21] ,Ingavle [22] , Deshawar [23].

Basic principles of electrospinning

Electrospinning is a straight forward easy method for obtaining sub-micron fiber diameters. In classical electrospinning the polymer melt or solution flows by means of a metering device through a nozzle which is often a blunt syringe needle connected to the positive terminal of a high voltage power supply. As the polymeric jet exits the nozzle it goes through a stretching process consist of two phases; steady state flow characterized by the straight flow of the jet and a chaotic phase characterized by a whipping swirl motion of the jet. The jet finally deposits on a collector connected to the ground terminal of the power supply

MODELING AND SIMULATIONCONCEPTS

Feng presented a model for electrospinning of non-Newtonian jet, in this model he stated that the exit boundary condition as follows:

“At the ‘‘exit,’’ we apply the asymptotic thinning conditions due to Kirichenko *et al.*[16] Specifically, as *R* and σ drop to zero and *E* approaches E_∞ , the stretching of the jet is governed by a balance among inertia, gravity, and tangential electrostatic force. This leads to the asymptotic scaling.

$R(z) \propto z^{-1/4}$Hence, we adopt the following exit conditions at $= \chi$:

$$R + 4zR' = 0 \qquad (1)$$

$$E = E_\infty \ (2)$$

According to the equation of electric field proposed by him which was derived by assuming the charge is distributed along a line and formulated as follows:

$$E(z) = E_{\infty}(z) - \ln\chi\left(\frac{1}{\epsilon}\frac{d(\sigma\ \ )}{d} - \frac{\beta}{2}\frac{d^2(ER^2)}{dz^2}\right) \quad (3)$$

This would be true only if R = 0 or $\sigma = 0$ but neither conditions is realistic because it is not possible for R to drop to zero in the straight jet region instead the jet has to go through the instability region for the radius to be assumed to drop to near zero and on the other hand σcannot be accepted to be assumed to drop to zero on exit because this contradicts with experimental observation by the limited ability to collect thick layers of deposited polymer mats due to the accumulation of residual surface charge

Based on this discussion there is a need to define a modified electric field equation and to introduce more realistic exit boundary conditions without depending on pre-assumptions of the relationship nature between Z and R

4.1 Derivation of modified electric field equation:

Let be assumed a finite element with cylindrical shape as shown in fig (1). If this element was charge with a surface charge q and according to gauss law which states " the net electric flux through a closed surface regardless its shape equals to the algebraic sum of each q_i charge located within the surface devided by ε_0 and to each q_s charge located on the surface devided by $2\varepsilon_0$

$$\phi = \oiint E \cdot n\, d \quad = \frac{1}{\varepsilon_0}\sum q_i + \frac{1}{2\varepsilon_0}\sum q_s \qquad (4)$$

Now assuming the Gaussian surface is coincident with the selected element surface

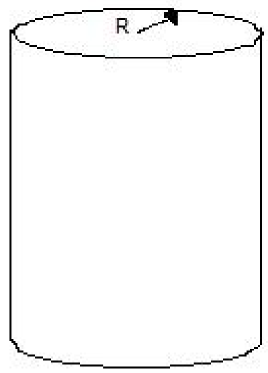


Figure.1: Elementary jet cylinder

$$\oiint E \cdot n\, d \quad = \frac{1}{\varepsilon_0}\sum 0 + \frac{1}{2\varepsilon_0}\sum q \qquad (5)$$

$$E_{n}^{l}\, dS_{n} + E_{t}^{l}\, dS_{tl} - E_{tl}^{l}\, dS_{tl} \; = \frac{q}{2\varepsilon_0} \quad (6)$$

$$E_r\ \ dS_r + d\left(E_z\ dS_z\right) = \frac{q}{2\varepsilon_0} \qquad (7)$$

$$E_r\ \ R + \frac{d}{2d}\left(E_z\ R^2\right) = \frac{q}{2\varepsilon_0 . 2\,\pi\, d} \qquad (8)$$

$$E_r\ \ R + \frac{d}{2d}\left(E_z\ R^2\right) = \frac{q\ .R}{2\varepsilon_0 . 2\,\pi\, .R\, .d} \quad (9)$$

By definition the surface charge density is given by:

$$\sigma = \frac{q}{S} = \frac{q}{2\pi.R.d} (10)$$

Substituting this relation gives:

$$E_r \ R + \frac{d}{2d}\left(E_z \ R^2\right) = \frac{\sigma.R}{2\varepsilon_0} \quad (11)$$

$$-E_r \ R = \frac{d}{2d}\left(E_z \ R^2\right) - \frac{\sigma.R}{2\varepsilon_0} (12)$$

It is known that the value of the electic potential at a point equals the algebric sum of all the existing components

Potential resulting from surface charge

$$V = \int -E_r \, dr - \int E_z \, dz \quad (13)$$

Assuming for convenience that

$$\int -E_r \, dr = V_r$$

and

$$-\int E_z \, dz = V_z$$

$$Vj = V_r + V_z \quad (14)$$

Potential resulting from external surrounding

$$V_e = V_\infty \quad (15)$$

Total voltage

$$V_t = V_\infty + Vj \quad (16)$$

$$V_t = V_\infty + V_r + V_z (17)$$

This would give

$$V_r = V_t - V_\infty - V_z (18)$$

But from above it is assumed that:

$$V_r = -\int E_r d \quad (19)$$

$$V_r = -E_r \, \Delta R \ (20)$$

$$V_r = -E_r (R - 0) \ (21)$$

$$V_r = -E_r \, . R \ (22)$$

$$-E_r \ R = V_t - V_\infty - V_z \quad (23)$$

$$\frac{d}{2d}\left(E_z \ R^2\right) - \frac{\sigma.R}{2\varepsilon_0} = V_t - V_\infty - V_z (24)$$

$$V_z = V_t \; - V_\infty - \frac{d}{2d}\left(E_z\, R^2\right) - \frac{\sigma.R}{2\varepsilon_0} \quad (25)$$

$$V_z = -\int E_z d \quad (26)$$

$$E_z = -\frac{d}{d} V_z \quad (27)$$

$$E_z = -\frac{d}{d}\left(V_t \; - V_\infty - \frac{d}{2d}\left(E_z\, R^2\right) - \frac{\sigma.R}{2\varepsilon_0}\right) \quad (28)$$

$$E_z = -\frac{dV_t}{d} - E_\infty - \frac{d2}{2dz^2}\left(E_z\, R^2\right) - \frac{d}{d}\left(\frac{\sigma.R}{2\varepsilon_0}\right) (29)$$

$$V_t = \Omega\, I \quad (30)$$

It was found experimentally that :

$$I \sim 10^{-9}$$

$$\Omega = \frac{1}{K} \sim 10^6$$

$$V_t = \Omega\, I \sim 10^6\, x\, 10^{-9} = 10^{-3} \cong 0$$

$$\frac{dV_t}{d} \cong \frac{d}{d}(0)$$

From the aforementioned it is concluded that the term

$$V_t = \Omega\, I \quad (31\,)$$

is small enough so that its influence on the behaviour of the electic field equation will be ignored in this work and will be studied in the future.
following this assumption the electric field equation becomes:

$$E_z = -E_\infty - \left(\frac{1}{2\varepsilon_0}\frac{d}{d}(\sigma.R) + \frac{1}{2}\frac{d2}{dz^2}\left(E_z R^2\right)\right) \quad (32)$$

Comparing this equation with Feng'sequation

$$E(z) = E_\infty(z) - \ln\chi\left(\frac{1}{\epsilon}\frac{d(\sigma\;\;)}{d} - \frac{\beta}{2}\frac{d^2(ER^2)}{dz^2}\right)(33)$$

the following is noticed

- The proposed equation does not depend on χ
- The proposed equation has all the main shape of Feng's equation but differs in the constants

4.2 Electric current density vector

$$J = J_{cc} \qquad + J_{cc} \qquad (34)$$

$$J = KE_l + \rho \quad (35)$$

$$\sigma 2\pi \;\; d = \rho \;\; R^2\, d$$

$$\rho = \frac{2}{R}\sigma \quad (36)$$

The electric current passing through the cross section of the jet is given as follows:

$$I = \iint J \cdot t_1 \, dA \qquad (37)$$

$$I = J_{t1} \cdot A$$

$$I = \left(KE_{t1}^{l} + \frac{2}{R} \sigma \, v \right) \pi \, R^2$$

$$I = (\pi \, R^2 KE_{t1}^{l} + 2\pi \quad \sigma \, v)$$

$$E_{t1}^{l} = E_{z}^{l}$$

$$I = \pi R^2 KE_{z}^{l} + 2\pi \quad v \, \sigma$$

4.3 Momentum equation

For one dimensional flow field

$$u_r = 0, \qquad u_6 = 0, \qquad u_z = v$$

$$\frac{1}{r}\frac{\partial(\, r u_r)}{\partial} = 0, \qquad \frac{\partial \quad _r}{r} = 0, \qquad \frac{\partial \quad _r}{\partial} = 0$$

$$\frac{1}{r}\frac{\partial(\, r u_6)}{\partial} = 0, \qquad \frac{\partial \quad _6}{r} = 0, \qquad \frac{\partial \quad _6}{\partial} = 0$$

$$\frac{\partial(u_z)}{\partial} = 0, \qquad \frac{\partial \quad _z}{r} = 0, \qquad \frac{\partial \quad _z}{\partial}$$

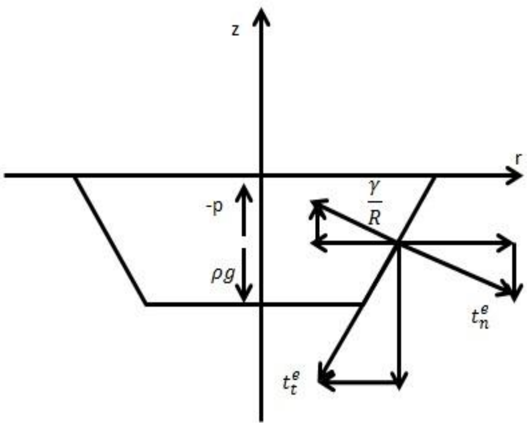


Figure.2: Force components acting on a finiteelement of the jet

Momentum equation in r direction yields

$$-p = \left(\frac{\gamma}{R} - (t_n^e - t_t^e R')\right) + o_r^p$$

Momentum equation in z direction yields after substituting the previous equation

$$\rho\,(vv') = -\rho \quad + \frac{\gamma R'}{R^2} - t_n^{e'} - t_n^e \frac{2R'}{R} + t_t^{e'} R' + t_t^e \left(R'' - \frac{2}{R}\right) + \frac{\partial(o_z^s)}{\partial} + \frac{\partial\left(o_z^p - o_r^p\right)}{\partial}$$

4.4 Giesekus model:

In this model it assumed that the total stresses the summation of two components

$$\sigma = \sigma^p + \sigma^s \quad (38)$$

$$o^p = o^e + o^v \quad (39)$$

$$o^s = -\eta_s \dot{\varepsilon} (40)$$

$$o^v = -\eta_p \dot{\varepsilon} \quad (41)$$

Where:

o : Total stress Tensor

The constitutive rheological equation for polymeric non-Newtonian liquids according to Giesekus model is given as follows:

$$o^p + \lambda_1 o_1^p + \left(\frac{\lambda_1 \alpha}{\eta_p}\right)(o^p \cdot o^p) = \eta_p \dot{\varepsilon} (42)$$

$$o_1^p = (U \cdot \nabla) o^p - (\nabla U)^T o^p - (o^p \cdot \nabla U) (43)$$

From analyzing the above equation, the following set is obtained

$$o_r^p + \lambda_1 \left(v \frac{\partial o_r^p}{\partial}\right) + \left(\frac{\lambda_1 \alpha}{\eta_p}\right)\left(o_r^p\right)^2 = 0 \qquad (44)$$

$$o_\theta^p + \lambda_1 \left(v \frac{\partial o_\theta^p}{\partial}\right) + \left(\frac{\lambda_1 \alpha}{\eta_p}\right)\left(o_\theta^p\right)^2 = 0 \qquad (45)$$

$$o_z^p + \lambda_1 \left(v \frac{\partial o_z^p}{\partial} - 2 \frac{\partial}{\partial} o_z^p\right) + \left(\frac{\lambda_1 \alpha}{\eta_p}\right)\left(o_z^p\right)^2 - 2\eta_p \frac{\partial}{\partial} = 0 \qquad (46)$$

Converting all the previous equations to the non-dimensional form utilizes the following non-dimensional numbers

4.5 Reference dimensions

Reference length

$$R_0 \quad (47)$$

Reference velocity

$$v_0 = \frac{Q}{\pi R_0^2} \quad (48)$$

Reference electric field

$$E_0 = \frac{I}{\pi R_0^2 K} \quad (49)$$

Reference Surface charge density

$$\sigma_0 = \bar{\varepsilon} E_0 \quad (50)$$

Reference stress

$$\tau_0 = m \quad ((\tau_0)_k , (\tau_0)_v) \ (51)$$

$(\tau_0)_k$: reference stress based on kinetic energy

$$(\tau_0)_k = \rho_0 v_0^2 \quad (52)$$

$(\tau_0)_v$: reference stress based on viscous stress

$$(\tau_0)_v = \frac{\eta_0 v_0}{R_0} \quad (53)$$

4.6 Non-dimensional groups

Electrical Peclet Number

$$P = \frac{2\bar{\varepsilon} v_0}{K R_0} \quad (54)$$

Froude: Number

$$F = \frac{v_0^2}{g R_0} \quad (55)$$

Reynolds Number

$$R = \frac{\rho v_0 R_0}{\eta} \ (56)$$

Weber Number:

$$W = \frac{\rho v_0^2 R_0}{\gamma} \ (57)$$

Electrostatic force parameter

$$\bar{\varepsilon}_1 = \bar{\varepsilon} \frac{E_0^2}{\rho v_0^2} \quad (58)$$

Ratio of dielectric constants

$$\beta = \frac{\varepsilon}{\bar{\varepsilon}} - 1 \quad (59)$$

Deborah Number

$$D = \frac{\lambda_1 v_0}{R_0} \ (60)$$

Relative Viscosity

$$\eta_r = \frac{\eta_p}{\eta_0} \quad (61)$$

In Giesekus Model $\eta_0 = \eta_p + \eta_s (62)$

As a result of the non-dimensional conversion the equation become in the form of first order Ordinary Differential Equations

For the ease of solving and treatment they are rewritten by substituting the following assumptions

$$y_1 = R_1 \quad (63)$$

$$y_2 = R_1^2 E_1 (64)$$

$$y_3 = R_1' (65)$$

$$y_4 = (R_1^2 E_1)' (66)$$

$$y_5 = \sigma_r^{p1} (67)$$

$$y_6 = \sigma_z^{p1} (68)$$

The final shape of Giesekus Model based equations

- **First equation**

$$y_1' = y_3 \qquad (69)$$

- **Second equations**

$$y_2' = \left[\frac{2\ P}{y_1^2}\left(E_{\alpha 1} - \frac{y_2}{y_1^2}\right) + \left(\frac{2y_3}{y_1} - \frac{2y_2 y_3}{y_1}\right)\right] \qquad (70)$$

- **Third equation**

$$\left[4\,\frac{1}{R}(1-\eta_r)\frac{1}{y_1^3}-\frac{\bar{\varepsilon}_1}{P}\frac{y_2}{y_1}(1-y_2)\right]y_3'$$

$$=\frac{1}{y_1^3}\left[\frac{2y_3}{y_1^2}-\frac{y_1^3}{F}+\frac{y_1y_3}{W}-\frac{\bar{\varepsilon}_1y_1^4}{P^2}(1-y_2)(y_3-y_4y_1-y_2y_3)-\beta\bar{\varepsilon}_1\frac{y_2}{y_1^2}(y_1y_4-2y_3y_2)\right.$$

$$-\bar{\varepsilon}_1\frac{y_3}{y_1^2}\left(\frac{y_1^6(1-y_2)^2}{Pe^2}+\beta y_2^2\right)$$

$$+\frac{\bar{\varepsilon}_1}{P}y_1y_3[y_2(y_3-y_4y_1-y_2y_3)+(1-y_2)(y_1y_4-2y_3y_2)]-\frac{2\bar{\varepsilon}_1}{P}y_1y_2(1-y_2)$$

$$+12\,\frac{1}{R}(1-\eta_r)\frac{y_3^2}{y_1}$$

$$\left.+\frac{\tau_0}{\rho v_0^2}\frac{y_1^5}{D}\left(\left(\frac{D}{\eta_r}\frac{\alpha\,R_0\tau_0}{v_0}\right)(y_5^2-y_6^2)+(y_5-y_5)-\left(2\eta_r\frac{\eta_0v_0}{R_0\tau_0}+2D\quad y_6\right)\frac{2\,y_3}{y_1^3}\right)\right]\quad(71)$$

- **Fourth equation**

$$y_4'=2E_{\alpha 1}+\frac{1}{P}(2y_1y_3-y_4y_1^2-2y_1y_2y_3)+\frac{2y_2}{y_1^2}\,(72)$$

- **Fifth equation**

$$y_5'=\frac{y_1^2}{D}\left(-\left(\frac{D}{\eta_r}\frac{\alpha\,R_0\tau_0}{v_0}\right)(y_5)^2-y_5\right)(73)$$

- **Sixth equation**

$$y_6'=\frac{y_1^2}{D}\left(-\left(\frac{D}{\eta_r}\frac{\alpha\,R_0\tau_0}{v_0}\right)(y_6)^2-y_6-\left(2\eta_r\frac{\eta_0v_0}{R_0\tau_0}+2D\quad y_6\right)\frac{2\,y_3}{y_1^3}\right)(74)$$

- **Initial conditions**

$$y_1^0=R_1\,(0)=1\quad(75)$$

$$\sigma_1^0=0\Rightarrow(1-y_2^0)\frac{y_1^0}{P}=0\quad(76)$$

$$\Rightarrow y_2^0=1$$

$$y_3^0=R_1'(0)=\tan\phi\,(77)$$

Assuming that:

$$\sigma_1'^0 = 0$$

$$\Rightarrow \quad \sigma_1'^0 = (y_3^0 - y_4^0 y_1^0 - y_2^0 y_3^0)\frac{1}{F}$$

$$\Rightarrow \quad y_4^0 = 0$$

It is assumed that the shear inside the nozzle is ineffective in stretching the molecules of the polymer in comparison with that which takes place outside the nozzle. This assumption means that the stresses at the nozzle are pure Newtonian and that $D = 0$ because $\lambda_1 = 0$ by substituting this in the non-dimensional form of Giesekus Model the following initial conditions are obtained

$$y_5^0 = \sigma_r^{p0} = 0$$

$$y_6^0 = \sigma_z^{p0} = 2\eta_r \frac{\eta_0 v_0}{R_0 \tau_0}\frac{\partial v_1}{\partial z_1} = 2\eta_r \frac{\eta_0 v_0}{R_0 \tau_0} v_1^{0\prime} = -4\eta_r \frac{\eta_0 v_0}{R_0 \tau_0}\frac{y_3^0}{y_1^{3\,0}}$$

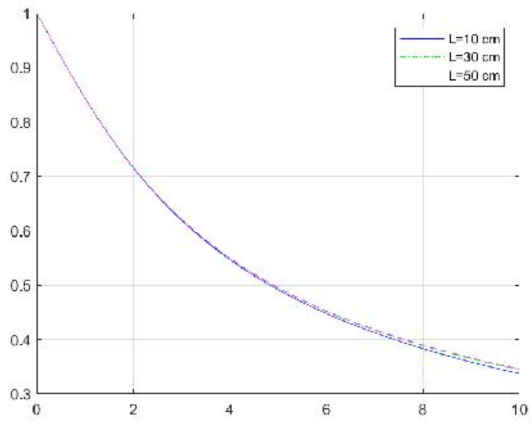
$$y_6 = -4\eta_r \frac{\eta_0 v_0}{R_0 \tau_0}\frac{R_1'(0)}{R_1^3(0)}$$

$$y_6 = -4\eta_r \frac{\eta_0 v_0}{R_0 \tau_0} R_1'(0) \quad (78)$$

Result and discussion

5.1 Influence of spinning distance

R1



Z1

Figure.3: Influence of spinning distance on the jet profile plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

It is noticed that as the spinning length increases the jet radius curve rises upwards and this could be explained by the reduction of the strength of the electric field resulting from the increase in distance between the source and the collector which in turn reduces the electric force exerted on the surface of the jet. This is physically represented as an increase in the jet radius

5.2 Influence of voltage

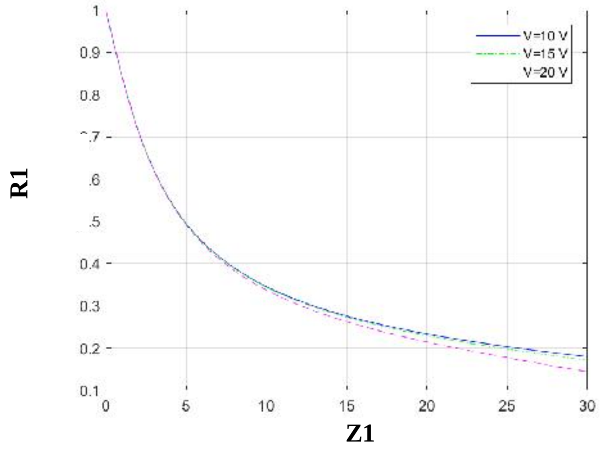


Figure.4: Influence of voltage on the jet profile plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

As the voltage increases the applied electric field increases which in turn increases the strength of the electric force leading to increased reduction in the jet radius

5.3 Influence of viscosity

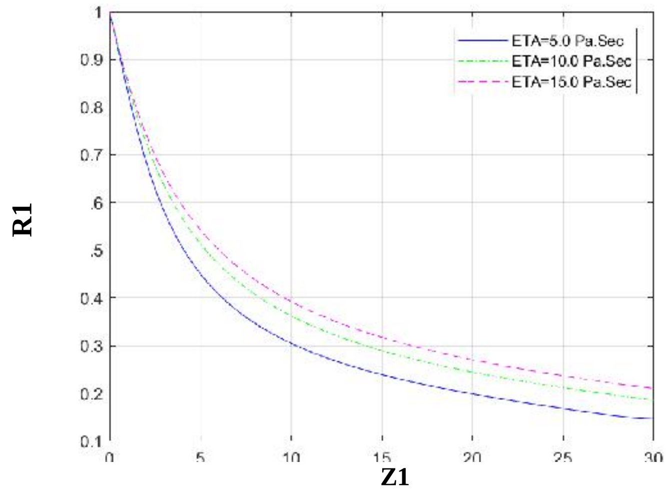


Figure.5: Influence of viscosity on the jet profile plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

It is noticed that as the viscosity of the jet solution increases the radius curve rises upwards and this could be explained by the increase in the viscous term of the forces acting on the jet which in turn means that the forces resisting the stretching increase leading to reduction in the jet radius

5.4 Influence of flow rate

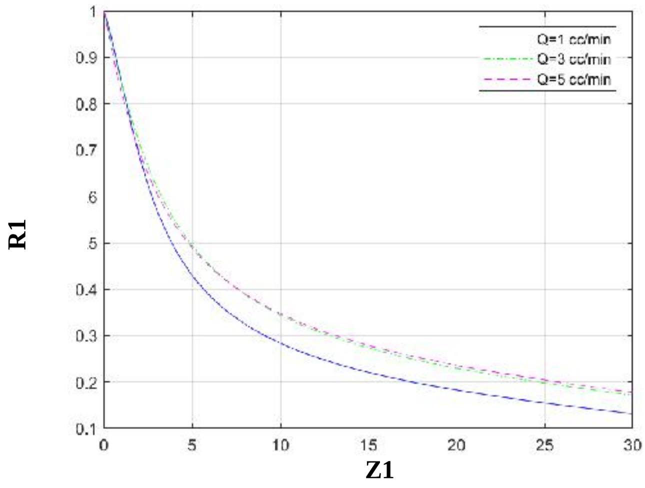


Figure.6: Influence of flow rate on the jet profile plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

It is noticed that as the flow rate increases with keeping the electric field and the spinning distance constant the only way for the jet to accommodate for the increase in its volume is to increase the radius of the jet

5.5 Fitting Theoretical curve

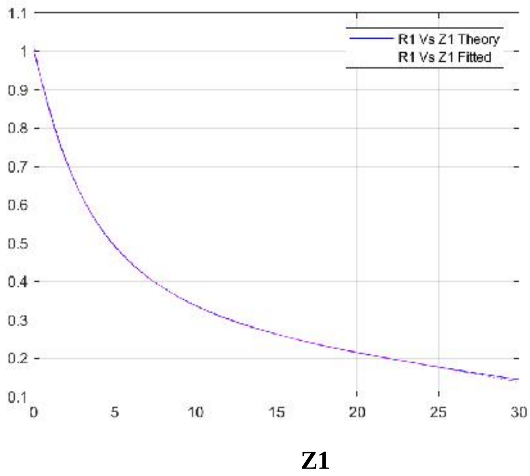


Figure.7: Comparison of Theoretical and fitted curves for selected parameters plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

The fitted curve was derived as a rational function to power 4 in the denominator

The fitting was conducted in Matlab 2017 and the following results were obtained
General model Rat04:

$$f(x) = (p_1)/(x^4 + q_1.x^3 + q_2.x^2 + q_3.x + q_4)$$

Coefficients (with 95% confidence bounds):

p_1 = 1.245e5 (1.242e5, 1.249e5)

q_1 = -33.65 (-33.68, -33.62)

q_2 = 107.8 (106.6, 109)

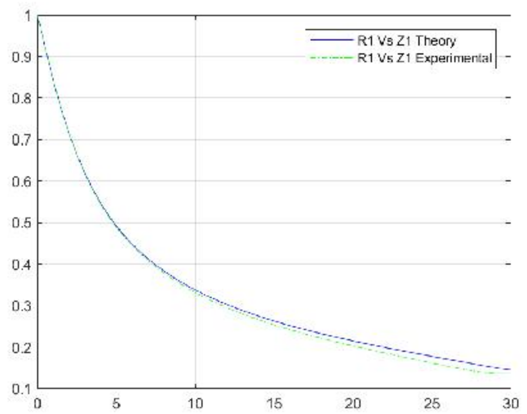
q_3 = 2.605e4 (2.597e4, 2.613e4)

q_4 = 1.226e5 (1.222e5, 1.229e5)

Goodness of fit:
SSE: 1.234
R-square: 0.9999
Adjusted R-square: 0.9999
RMSE: 0.001787

As a result it is concluded that the assumption of Feng that $R(z) \propto z^{-1/4}$ holds in the solution obtained by this study

5.6 Compare Theoretical curve to Experimental



Z1

Figure.8: Comparison of Theoretical and Experimental curves for the above selected parameters plotted as Non-dimensional radius of the jet versus the non-dimensional distance between source and collector

It is observed from the two curves that they are close to each other yet the experimental one lies below the theoretical curve and this could be explained by the reduction in the jet radius due to evaporation of the solution

Conclusion

A modified equation for the electric field has been derived and applied along with the rest sets of ODE equations and solved using MATLAB 2017

The terminal radius of the jet increases with the increase of volumetric flow, , solution viscosity and source radius while decreases with the increase of, Voltage, spinning distance. From the plots it is possible to notice that the accuracy of the proposed model is 93%

Nomenclature

M	Mass
P	Density
V	Volume
u_r	Velocity in r direction
R	Radius of Jet
u_θ	Velocity in θ direction
u_z	Velocity in Z direction
$F^g{}_r$	Force of Gravity in r direction
$F^\gamma{}_r$	Force of surface tension in r direction
$F^e{}_r$	Force of electric field
$F^\eta{}_r$	Force of viscosity in r direction
J	Electric charge density
σ^s	Solution stress
σ^e	The elastic component of the polymer stress tensor
σ^v	The viscous component of the polymer stress tensor
$\sigma^p{}_1$	Upper time convicted derivative of the polymer stress
λ_1	Relaxation time
α	Mobility factor
η_p	Polymer viscosity
$\dot{\varepsilon}$	Rate of strain
U	Velocity vector
$t^e{}_n$	Normal stress vector
$t^e{}_t$	Tangential stress vector
E_t	The tangential stress vector of the electric field
E_n	The normal stress vector of the electric field
v_0	Reference velocity
K	Electric conductivity
E^l	The electric field vector
R_0	Initial radius of the jet
γ	Surface tension
E_0	Reference electric field
ε	Dielectric constant of the jet
η_0	Overall viscosity

R_1	Non-dimensional radius of jet
E_1	Non-dimensional electric field

References

[1] Gray S A, Letter concerning the electricity of water, from Mr. Stephen Gray to Cromwell Mortimer, M.D.Secr.RS *Phil. Trans*, 1731-1732.37227, 1753- http://dx.doi.org/10.1098/rstl.1731.0040
[2] Larmor J, Note on the complete scheme of electrodynamic equations of a moving material medium, and on electrostriction *Proc. R. Soc*, 1898. 63 365
[3] Sze, S. Y., SU, L. K, *Effect of an electric lens on water jets,* The American Physics Teacher, September 1936,4, 139
[4] Cooley J F, *Apparatus for electrically dispersing fluids* US Patent Specification 692631, 1902
[5] Morton W J, *Method of dispersing fluids* US Patent Specification 705691, 1902
[6] Formhals A, Process and apparatus for preparing artificial threads *US Patent Specification* 1975504, 1934
[7] Formhals A, Artificial fibre construction *US Patent Specification 2109333*, 1938a
[8] Forrnhals A, Method and apparatus for the production of fibres*US Patent Specification* 2116942, 1938b
[9] Forrnhals A, Method and apparatus for the production of fibres*US Patent Specification* 2123992, 1938c
[10] Forrnhals A, Method and apparatus for the production of artificial fibres*US Patent Specification* 2158416, 1939a
[11] Formhals A, Method and apparatus for spinning *US Patent Specification* 2160962, 1939b
[12] Zeleny*I*, Instability of electrified liquid surfaces *Phys. Rev*, 1917. 10 1,
[13] Taylor G, Disintegration 0: water drops in an electric field *Proc. R. Soc, 1964.* A 280383,
[14] Taylor G, Studies in electrohydrodynamics: L The circulation produced in a drop by electrical field *Proc. R. Soc*, 1966. A 291145
[15] Taylor G Electrically driven jets *Proc. R. Soc*, 1969. A 313 453
[16] Spivak A., Dzenis Y., Reneker D., "A Model of Steady State Jet In The Electrospinning Process", Mechanics Research Communications, 27 (1), 37-42 (2000).
[17] Spivak A., Dzenis Y., "Asymptotic decay of radius of a weakly conductive viscous jet in an external electric field", Applied Physics Letters, 73 (21), 3067-3069 (1998).
[18] Hohman M., Shin M., Rutledge G., Brenner M., "Electrospinning and electrically forced jets. I. Stability theory", Physics Of Fluids, 13 (8), 2201-2220 (2001).
[19] Hohman M., Shin M., Rutledge G., Brenner M., "Electrospinning and electrically forced jets. II. Applications", Physics of Fluids, 13 (8), 2221-2236 (2001).
[20] Feng J., "The stretching of an electrified non-Newtonian jet: A model for electrospinning", Physics of Fluids, 14 (11), 3912-3926 (2002).
[21] Arumuganathar, S., Jayashinghe, S.N., Living scaffolds (specialized and unspecialized) for regenerative and therapeutic medicine. Biomacromolecules 9, 759 (2008).
[22] Ingavle GC, Leach JK., Advancements in electrospinning of polymeric nanofibrous scaffolds for tissue engineering, Tissue Eng Part B Rev. Aug;20(4):277-93 (2014).
[23] Deshawar D, ChokshiP,Stability analysis of an electrospinning jet of polymeric fluids,Polymer, 131, 34-49, 2017.