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Estimating the Parameters of GP Model and AR model using Gibbs Sampler for Spatio-Temporal Datawith Comparison Study

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ABSTRACT: *In the last 2-decades, more attention has been paid to the analysis and modeling of complex stochastic systems of large database, particularly time-spatial data. This paper presents models which help researchers to represent temporal-spatio data, the most important models are GP and AR. It also illustrates the specifications of these models related to the data. was Bayes hierarchical method is used to divide these two models to observation and structure equations.*
This paper provides comparison study between Gaussian process (GP) and auto-regressive (AR) models to a database of pollution measurements for five stations in Aleppo city-Syria, special attention is given to studying of carbon dioxide (CO), as well as investigating the side effect of covariates on the ratio of air pollution, it has been found the environment temperature has a great effect on the increase of CO ratio in the air. A data-Base records for 2-months has been acquired for the 5-station installed in Aleppo area, this procedure is attended to be capable to determine the fitting model, the Data-Base of last two days of September in 2009 were used for the temporal forecasts. Three statistical criteria; the root mean square error (RMSE), mean absolute percentage error (MAPE) and bias (BIAS), used to examine the validation of the studied models for being valid for forecasting. The statistical analysis showed that the performance of the Gaussian model is superior to the auto-regressive model with the average of RMSE = 0.032 and 0.067, MAPE = 1.935 and 2.515, and BIAS = -0.015 and -0.067, for GP and AR, respectively.

Keywords:*Bayesian inference, Hierarchical Bayesian Spatio Temporal Model, GP, AR, MCMC Technique, GSA.*

INTRODUCTION

In nature and technical applications there are many events and phenomena, that one can model them mathematically, for better understanding and description of the underlying processes. The model might be bused on point groups of time independent, spatial events or temporal-spatial events [1][2].

Most of the recent researches and studies aimed at using the Bayes hierarchical models, due to their high efficiency. They give accurate results in the analysis of their associated data. Academics showed interest in this type of modeling due to the difficulty of dealing with linear models in the analysis and prediction of the many constraints and assumptions. As well as the emergence of many problems i.e. economic and biological problems that are not suited to linear models has led to using Bayesian hierarchical models[3].

This paper introduces a short overview of the Bayesian and Bayesian hierarchical approach. In addition, it provides a brief review of the models used for spatio-temporal data. It illustrates the models through real-life practical data applications using R-spTimer package, as well as providesthe pseudo code for steps of statistical analysis (i.e. the training dataset for modelling and forecasting, which is distributed in both space and time) ofHierarchical Bayesian Spatio-Temporal Model(HBST) models applied to air pollution measurements database.

STANDARD BAYES APPROACH [4]

Bayesian methods are commonly used in estimation and inference statistical. This principle was introduced by Thomas Bayes in 1967.Bayesian modeling generally depends on the use of prior information about the unknown parameters$\theta = (\theta_1, \theta_2, \dots, \theta_n)$,Which require estimation as these parameters are random variables and not fixed values, where this information is in the form of a preliminary probability function. This function is called prior distribution, which is denoted by $\pi(\theta)$, i.e. before sampling. After obtaining the sample, the probability distribution function for the current sample observations depends on the parameters θ, denoted by $f(\underline{X}|\theta)$,it is called the likelihood function. the posterior distribution is obtained using Bayes theory as follows, is given by:

$$\pi(\theta|\underline{X}) = \frac{f(\underline{X}|\theta)\pi(\theta)}{f(\underline{X})} \tag{1}$$

$f(\underline{X})$can be entered into the proportionality constant, i.e. there is no need for calculation. The posterior distribution $\pi(\theta|\underline{X})$becomes the following:

$$\pi(\theta|\underline{X}) \propto f(\underline{X}|\theta)\pi(\theta) \tag{2}$$

HIERARCHICAL BAYESIAN SPATIO-TEMPORAL MODEL (HBST)

Hierarchical Bayesian Spatio Temporal (HBST) model is a statistical modeling technique that deals with space-time modeling in Bayesian approach.

The general concept of the hierarchical expression means that the specifications of the models are divided into levels or stages. This division illustrates the specifications of any model by collecting syntactic information, in particular quantitative information. Another concept can be presented to hierarchical models. The data in the first stage depend on some parameters and are called the First stage parameters, these parameters in turn depend on the parameters of the second stage and so on. This model is designed to deal with complex models, i.e. instead of being in right decision, it works to make better decision. It is more accurate than linear models in dealing with social, biological, environmental and engineering problems ,as well as provide prediction accuracy of the results obtained [5].

Gelfand 2012 suggests that the spatial-temporal models of Bayes should be represented by three stages of hierarchical structure as follows [6]:

Stage 1: Data Model$[Z|O, \theta]$
stage 2: Process Model $[O|\theta]$
stage 3: Parameter Model$[\theta]$

where$Z$ is the data, the (hidden) process is defined by $\boldsymbol{O}$, and the unknown parameters are specified by θ.

Suppose that $Z_l(s_i, t)$denotes the observed point-referenced data where $l = 1, \dots, r$ denotes longer-term (years) and $t = 1, \dots, T_l$denotes shorter-term time (Days) and $s_i$denotes the location

$i = 1, \dots, n$.Suppose that $O_l(s_i, t)$is the true value corresponding to $Z_l(s_i, t)$ ats_i. Let $Z_{l_i} = (Z_l(s_1, t), \dots, Z_l(s_n, t))^T$and $O_{l_i} = (O_l(s_1, t), \dots, O_l(s_n, t))^T$.

3.1 Bayesian Gaussian Process Model

Cressie and Wikle 2011 define the specifications of the Gaussian model as it consists of two equations; theobservation equation which isthe interpretation of the equation as a linear model interpreted and second one is the structural equation, which describes the relationship between the parameters of the first stage and the set of new parameters, as a function of time. In this paper the true process was modeled as a function of covariates X. the hierarchical structure is given as follows[7]:

$$Z_{l_i} = O_{l_i} + \epsilon_{l_i} \qquad (3)$$

$$O_{l_i} = X_{l_i}\beta + \eta_{l_i} \qquad (4)$$

For each $l = 1, \dots, r$ and$t = 1, \dots, T_l$
Where:
$\boldsymbol{O_{l_i}}$:the true process corresponding toZ_{l_i}
$\boldsymbol{\epsilon_{l_i}}$: the pure error term assumed to be independently normally distributed.
$\epsilon_{l_i} = (\epsilon_l(s_1, t), \dots, \epsilon_l(s_1, t))^T \sim \mathrm{N}(0, \sigma_\epsilon^2 \mathrm{I}_n)$ is a nugget effect.
$\boldsymbol{\beta}$is the p×1vector of regression coefficients of X_{l_i}, where $\beta = (\beta_1, \dots, \beta_p)$
$\boldsymbol{\eta_{l_i}}$**:**is the Spatio-temporal random effect, where $\eta_{l_i} = (\eta_l(s_1, t), \dots, \eta_l(s_n, t))^T$, and these will be assumed to follow $\mathrm{N}(0, \Sigma_\eta)$ independently in time.$\Sigma_\eta = \sigma_\eta^2 S_\eta$, $\sigma_\eta^2$is the site invariant spatial variance and $S_\eta = \kappa(\mathrm{si}, \mathrm{sj}; \phi, \upsilon)$ is the spatial correlation matrix that have spatial decay ϕ and smoothness parameter υ. Both parameters, ϵ_{l_i} and η_{l_i} are independent each other.
$\boldsymbol{X_{l_i}}$: is denoted all the n×p covariates matrix.
θsymbol is used to assert all parameters usedθ = $\left[\beta, \sigma^2{}_\epsilon, \sigma^2{}_\eta, \phi, \nu\right]^T$,sampling parameters ϕ a νcorresponding to the two different prior distributions, only allowed for the decay parameter ϕ is to assign a continuous uniform prior distribution over an interval or a Gamma prior distribution.

3.2 Bayesian Auto-Regressive Model

Sahuet al. (2007), the hierarchical AR models are defined as follows [8]:

$$Z_{l_i} = O_{l_i} + \epsilon_{l_i} \qquad (5)$$

$$O_{l_i} = \rho O_{l_i - 1} + X_{l_i}\beta + \eta_{l_i} \qquad (6)$$

whereϵ_{l_i} and η_{l_i} form is defined as in the previous GP model. The ρ denotes the autoregressive process parameter, assumed to be in the interval[-1,1]. In the particular case where ρ = 0,the AR model turns to GP one.

In Bayesian analysis, note that complex hierarchical models often require the integration of functions of multiple dimensions, which are often analytically intractable and are hard to fit. Therefore, many of the abbreviated methods have been proposed by researchers such as Smith 1991, Evans and Swartz 1995 for evaluating features of posterior distributions needed for making inference, one of them is following *Markov chain Monte Carlo* methods.

MARKOV CHAIN MONTE CARLO METHODS [9] [10]

Markov Chain Monte Carlo (MCMC) techniques are methods for sampling from probability distributions using Markov chains. It is one of the most widely used simulation methods. Most of the researches and recently have taken another trend in the calculation of integrals when finding the (posterior)marginal densities of the parameters. This method depends on the generation of a series of values from a (posterior) distribution using primary values, these values can be used to approximately calculate the estimations of different parameters. One of the most famous methods of Markov chainsis Gibbs Sampler algorithm (GSA).

4.1 Gibbs Sampler Algorithm (GSA)

Is a technique for the generation of random variables of marginal distributions that were introduced by the Geman and Geman in 1984 and originally derived from the algorithm of Metropolis and Rosenblath (1953) and has been developed by Gelfand and Smith (1990).The origin of the GS Aappeared in the subject of image processing as it has common application and is widely applied in the solution of Bayes problems. It has shown great potential in physical and engineering research to obtain complex integrals calculations by expressing these integrations with projections of some probability distributions and then estimating this projection by sampling of that distribution. Addition to all the models are fitted using GSA [9].

The primary key to GSA is to know the one-variable conditional distributions because application of the simulation method is easier than applying to the common distributions. To find marginal distributions using GSA, one has follow these steps.

Steps of GSA [10]:

1- Suppose there are the following random variables $[\theta_1, \theta_2, \dots, \theta_k]^T$and the following starting initial values$[{\theta_1}^{(0)}, {\theta_2}^{(0)}, \dots, {\theta_k}^{(0)}]$
2- Draw a value ${\theta_1}^{(1)}$ from the full conditional $[\theta_1 | {\theta_2}^{(0)}, {\theta_3}^{(0)}, \dots, {\theta_k}^{(0)}]$
3- Draw a value ${\theta_2}^{(1)}$ from the full conditional $[\theta_2 | {\theta_1}^{(0)}, {\theta_3}^{(0)}, \dots, {\theta_k}^{(0)}]$
4- Draw a value ${\theta_k}^{(1)}$ from the full conditional $[\theta_k | {\theta_2}^{(0)}, {\theta_3}^{(0)}, \dots, {\theta_{k-1}}^{(0)}]$ raw $\theta^{(2)}$ using $\theta^{(1)}$and continually using the most updatedvalues.
5- Repeat until are obtained M draws, with each draw being a vector $\theta^{(j)}$, with j iteration.
6- Optional burn-in and/or thinning.

PRACTICAL APPLICATION

There is an increasing interest in the field of environmental sciences to model climate data measured using monitoring stations located it different sites in the area under investigation. One of the modeling objectives is to obtain spatial predictions at a location where observations are missed and obtain temporal predictions of future time points. The main goal in this application is to find a fitting model for temporal-spatial data studied using Bayesian modeling, as well as estimating its parameters using GSA.

The study was conducted on data representing measurements of air pollution for five stations in Aleppo city. The measurement was taken during the period from 1/1/2009 to 31/5/2011.Special attention is given to studying of carbon dioxide (CO) for the months of August and September in 2009.Four important climate covariates were considered: ultraviolet B (UVB in watts per square meter), temperature (Tempin degree Celsius), wind speed (wind.spd in nautical miles) and relative humidity (RH), and to examine the effect of these variables on increasing the level of carbon dioxide in the air. This is known through the linear part of the model on the parameters of regression β, and the capabilities of both the Gaussian model and the auto-regressive were found using the GSA, provided that the Acceptance Rate of the parameter is set between 15% and 40% and the appropriate distribution of this parameter is either continuous or discontinuous distribution or gamma distribution [11]

Table 1, shows the Pseudo code for steps of statistical analysis ofHBST models applied to pollution measurement database, using spTimer-R [11].

Table 1. Pseudo Code for steps of statistical analysis of HBST models

Start:

Data-Base:

$Z = Z(s_1, t), \ldots, Z(s_5, t)$ are a values of the carbon dioxide (CO) for each $t = 1, \ldots, T$

$O = (O(s_1, t), \ldots, O(s_5, t))^T$: are true values corresponding to Z

$X = [X_1, X_2, X_3, X_4]^T$: are 4 covariates (UVB, Temp, wind.spd and RH)

T is set at 29 at this work to suit selected data from 1-30 of August and September in 2009 as the model input

We put 30,31of September in 2009 as our forecasting target.

Model Fitting:

1- sampling parameters ϕ : using R, Gamma distribution (9,3) was most appropriate for this parameter corresponding to acceptance rate 38.54% for the AR model and 39.86% for the GP model [11] [12].

2- Apply the Gaussian Process Model to the dataset:

3- Estimate the parameters $\left(\boldsymbol{\theta} = \left(\boldsymbol{\beta}, \boldsymbol{\sigma}^2_{\epsilon}, \boldsymbol{\sigma}^2_{\eta}, \boldsymbol{\phi}, \boldsymbol{\nu}\right)\right)$ of the model using GSA.

4- Apply the Auto-Regressive Model to the dataset:

$$Z_{l_i} = O_{l_i} + \epsilon_{l_i}$$

5- Estimate the parameters of AR model using GSA.

6- ComputePMCC using [13]:

$$\boldsymbol{P} = \sum_{i=1}^{n} \sum_{l=1}^{r} \sum_{t=1}^{T_i} \left\{ \boldsymbol{E}\left(\boldsymbol{Z}_l(\boldsymbol{s}_i, \boldsymbol{t})_r - \boldsymbol{z}_l(\boldsymbol{s}_i, \boldsymbol{t})\right)^2 + \boldsymbol{V}\left(\boldsymbol{Z}_l(\boldsymbol{s}_i, \boldsymbol{t})_r\right) \right\} \quad (7)$$

where $Z_l(s_i, t)_r$ denotes a future replicate of the data $z_l(s_i, t)$.

the smaller value of PMCC criteria determines the best model fit.

Validationand Prediction:

1- For validation purpose, Compute the statistical criteria [14]:

$$M = \frac{1}{m} \sum_{i=1}^{m} (\hat{z}_i - z_i)^2 \quad (8)$$

$$M = \frac{1}{m} \sum_{i-1}^{m} \left| \frac{\hat{z}_i - z_i}{z_i} \right| \quad (9)$$

$$B = \hat{z}_i - z_i \quad (10)$$

where, m is the total number of observations we want to validate, z_i is the data indexed by i, $\hat{z}_i$ is the prediction value.

the Smaller parameter values determine the best predictive model.

2- For forecasting purpose, the HBST GP model at any observed point s_i on T+1 day is expressed as follows [15]:

3- using Monte Carlo Markov Chain (MCMC)-Gibbs sampling method to predict $Z_{l_i}(s_i, T+1)$ with j iterations, according to the following algorithm [15] [16]:

3.1. Draw a sample $\theta^{(J)}$ and $O^{(J)}, j \geq 1$ from $\pi(Z(s_i, T+1)|z)$

3.2. Draw $O_{l_i}{}^{(J)}(s_i, T+1)$ from $N\left(X^{(J)}{}_{l_i}\beta^{(J)}, \sigma_j^{2(J)}\right)$

3.3. Finally draw $Z_{l_i}{}^{(J)}(s_i, T+1)$ from $N(O_{l_i}{}^{(J)}(s_i, T+1), \sigma_\epsilon^{2(J)})$

Where posterior predictive distribution of $Z_{l_i}(s_i, T+1)$ given z done using:

$$\pi(Z_{l_i}(s_i, T+1)|z) = \int \pi(Z_{l_i}(s_i, T+1)|\theta, O, O(s_i, T+1), z)\, \pi(O(s_i, T+1)|\theta, z)\, \pi(\theta, O|z)\, d\ (s_i, T+1)\, d\ \ d \qquad (13)$$

Results and Discussion

This section provides a comparison study and prediction results of two models. For purpose, to apply steps of pseudo code of HBST model we used spTimer package [11]. The pollution measurement database is divided into three parts, one for model fitting, the other two are for validation and prediction.The GP model described in eq. (3) and (4) and AR model described in eq. (5) and (6) are fitted. We randomly selected 3 locations and 2 months of data as a model fitting, i.e.3*2*29=116 observations are used. From R-SpTimer, we obtained the summary statistics of the model parameters as following:

For AR model

```
-----------------------------------------------------
Model: AR
Call: CO ~ UVB + Temp + RH + Wind.spd
Iterations: 5000
nBurn: 1000
Acceptance rate for phi (%): 39.86
-----------------------------------------------------
Goodness.of.fit Penalty   PMCC
values:       0.14    22.96    23.1
-----------------------------------------------------
Computation time: 4.54 - Sec.
```

For GP model

```
-----------------------------------------------------
Model: GP
Call: CO ~ UVB + Temp + RH + Wind.spd
Iterations: 5000
nBurn: 1000
Acceptance rate for phi (%): 38.54
-----------------------------------------------------
Goodness.of.fit Penalty   PMCC
values:           0.20    25.42    25.62
-----------------------------------------------------
Computation time: 5.12 - Sec.
-----------------------------------------------------
```

Table (1) shows comparison between GP and AR models using PMCC measure, note that the AR model is the most fitting for our data compared is GP model:

Table 1.comparison between GP and AR models using PMCC measure

	Goodness of fit	Penalty	PMCC
Model AR	**0.14**	**22.96**	**23.10**
Model GP	**0.20**	**25.42**	**25.62**

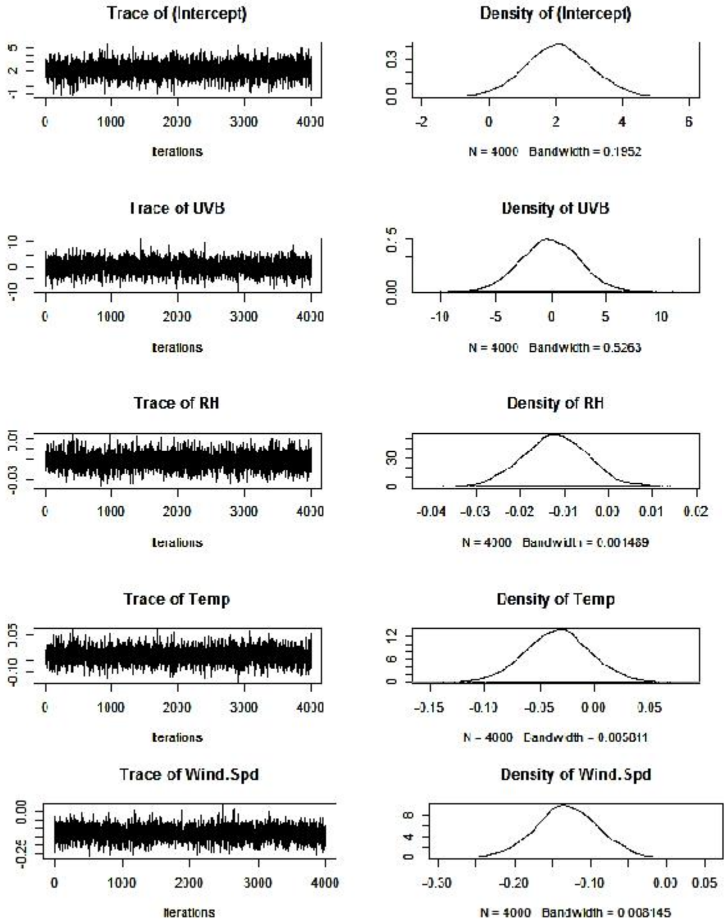

Figure.1:MCMC trace plots for the four model parameters for 5000 iterations

Figure 1 shows the MCMC trace plots for the four parameters for 5000 iterations, these graphs represent the regression parameters values for all the UVB, RH, Temp, and Wind.spd covariates with the number of iterations. We note that the first 1000 samples are discarded as burn-in, i.e. this helps to stationary the series and the approximation of the distribution of parameters from normal distribution.
Figure 2(a), shows the Normal Q-Q plot, it is observed the random spreading of residuals around the line. Figure 2(b) shows that there is no spreading of residuals according to a specific direction, i.e. the residuals follow the normal distribution. These two Figures support our decision that theAR model is fitting the data under investigation.

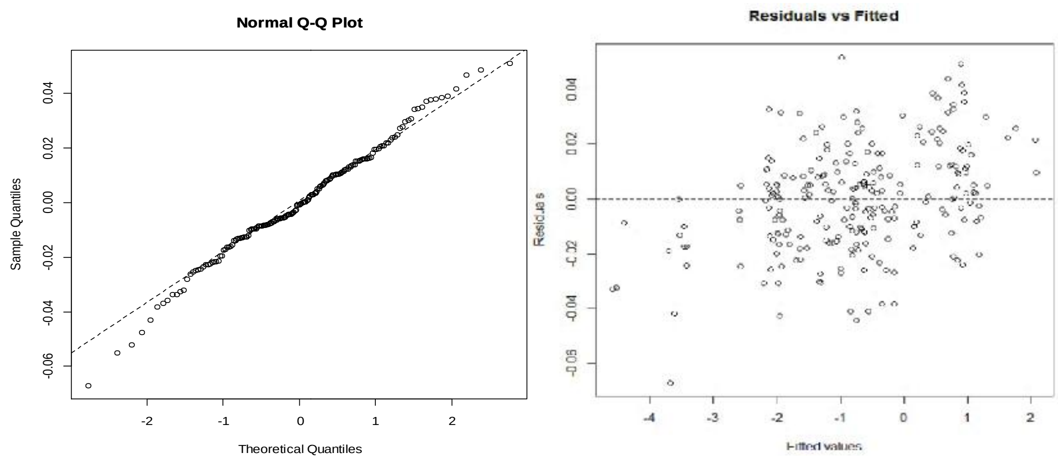

Figure.2:Normal Q-Q plot for (a), spread of residuals for (b)

Table (2) represents the values of the parameter estimates and the descriptive statistics, i.e. mean, median, standard deviation, and 95% confidence intervals for each estimator for the AR model:

Table 2.parameter estimates from the MCMC samples

	Mean	Median	SD	Low2.5p	Up97.5p
(Intercept)	**0.0449**	**0.0402**	**0.4581**	**-0.8390**	**0.9756**
UVB	**-0.7693**	**-0.7676**	**1.1139**	**-2.9907**	**1.3988**
Temp	**0.0128**	**0.0130**	**0.0153**	**-0.0175**	**0.0418**
RH	**-0.0027**	**-0.0027**	**0.0035**	**-0.0098**	**0.0044**
Wind.spd	**-0.1309**	**-0.1317**	**0.0410**	**-0.2121**	**-0.0514**
ρ	**0.8028**	**0.8031**	**0.0448**	**0.7119**	**0.8880**
σ^2_{ϵ}	**0.0161**	**0.0160**	**0.0018**	**0.0130**	**0.0200**
σ^2_{η}	**0.3097**	**0.3065**	**0.0357**	**0.2474**	**0.3880**
□	**3.0252**	**2.8926**	**1.0076**	**1.3941**	**5.3536**

Note that the covariates UVB, RH and Wind.spd are statistically insignificant (i.e. have no effect on increasing CO concentration in the air) except for the estimate of the variable temperature. It is observed that it has an effect of 0.0128, i.e. anincreasein temperature of one degree will resultsinincreasing ofCOwith concentration of 0.0128 in the air.Note that the spatial variance$\sigma^2_{\eta} = 0.306$is also called sill (roughly the boundary between it and the predicted values) is higher than the pure error variance $\sigma^2_{\epsilon} = 0.0160$, which is called Also with varying white noise. As for □ indicates a small spatial correlation because: $\frac{-\text{l}\ (0.0\ )}{2.8} = 0.45\,K$,the effective range is very small within a distance of 0.45 km. We have the value of the temporal auto-regressive parameter ρ = 0.8031 and this indicates that the current time period is strongly correlated with the previous time period for the measurements of carbon monoxide gas.

Finally, after estimating and interpreting the parameters of the model, GP model that represents the database is give as follows:

$$O_{l_i} = 0.8031 O_{l_i-1} + 0.0449 - 0.7693 X_1 + 0.0128 X_2 - 0.0027 X_3 - 0.1309 X_4$$

Whereρ $= 0.8031$,X_1 is UVB variable, X_2 is the Temp variable, $X_3$is the wind.spd variable and X_4 is the RH variable, regression coefficients are:$\beta_0 = 0.0449$, $\beta_1 = 0.7693$, $\beta_3 = 0.0027$, and $\beta_4 = 0.1309$.

Finally, to verify the validation of the model, statistical analysis was performed through three validation criteria: the root mean square error (RMSE), mean absolute percentage error (MAPE) and bias (BIAS).Table (3), shows the results of validation process, it is seen the fitted GP model performs much better than the fitted AR model.

Table 3. Validation statistics for the GP and the AR models

	MSE	**MAPE**	**BIAS**
Model GP	**0.0018**	**1.9354%**	**-0.0150**
Model AR	**0.0046**	**2.5154%**	**-0.0131**

It is observed from the MAPE scale that the error rate in prediction is very small and this indicates a very high reliability of prediction 98.06% for the GP model and 97.48% for the AR model, noted that less accurate of AR model due to the AR temporal correlation parameter ρ.

On the MSE scale, we notice that the error values in the GP model are lower than the values in the AR model,thenthe statistical analysis showed that the performance of the Gaussian process model was better than the auto-regressive model.

the Data-Base of last two days of September 2009 were used the temporal forecasts(Two-step ahead forecast, i.e., in day 61 and 62).Figure 3, shows the forecast values with credible 95% interval for GP spatio-temporal model.

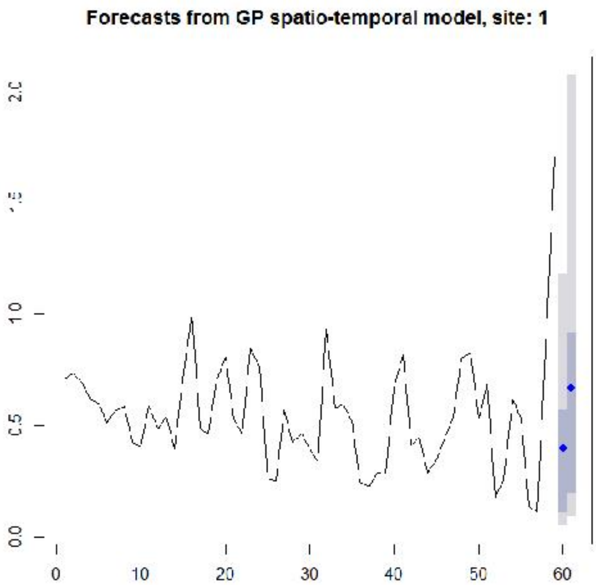

Figure.3: 2-dayes ahead forecast in original scale.

CONCLUSION

In this paper, the most important types of modeling for temporally-spatially distributed data are Bayesian modeling, which helps to clarify the model's specifications and to show all the parameters related to time, space and space-time. Then comparison study between GP and AR assisted fitting and predication for space-time data using Bayesian hierarchical model has been discussed. This work has been successfully compared to two HBST models to fit and forecast air pollution measurements distribution the most important results were reached:

1- The effectiveness of Bayes' hierarchical method of describing and modeling temporally-spatially distributed data and its large role to accommodate such data.
2- The ability of the AR model to fit the data more than the GP model using the PMCC standard.
3- The HBST-GP model was successful to perform the forecasting of air pollution measurements over the Aleppo region.
4- The GP obtained 98.06% for forecasting accuracy weather the AR obtained 97.48%.

suggested to apply at the future work using more data from air pollution measurements, Work on the modelling of temporal-spatial data using fuzzy logic to Processing of uncertainty process.

REFERENCES

[1] D.J Daley, D. Vere-Jones, An Introduction to the Theory of Point Processes. Volume I: Elementary Theory and Methods, 2nd edition. Springer-Verlag, New York, 2003.
[2] Baddeley, A.: Spatial Point Processes and their Applications. Australia, 2005.
[3] Draper, D., (1995), “Inference and Hierarchical Modeling in the Social Sciences”, J.R.Statist.Soc.,Series A,156,pp. 9-38
[4] Box, G.E. and Tiao, G.C., (1973), “Bayesian Inference In Statistical Analysis”, Addison – Wesley Publishing Company, London.
[5] Sahu S K, Bakar K S, and Awang N 2013 Bayesian forecasting using hierarchical spatiotemporal models with applications to ozone levels in the eastern united states *Technical Report* School of Mathematics University of Southampton Southampton.
[6] Gelfand AE (2012). "Hierarchical Modeling for Spatial Data Problems." Spatial Statistics, 1,30-39.
[7] Cressie NAC, Wikle CK (2011). Statistics for Spatio-Temporal Data. John Wiley & Sons, New York.
[8] Sahu SK, Gelfand AE, Holland DM (2007). "High-Resolution Space-Time Ozone Modeling for Assessing Trends." Journal of the American Statistical Association, 102, 1221-1234.
[9] Gelfand AE, Smith AFM (1990). "Sampling-Based Approaches to Calculating Marginal Densities." Journal of the American Statistical Association, 85(410), 398{409.
[10] KhandokerShuvoBakar, M.S. & B.Sc. " Bayesian Analysis of Daily Maximum Ozone Levels". University of Southampton, PhD Thesis, pagination, January 2012.
[11] K.S Bakar, Sujit K. Sahu, "spTimer: Spatio-Temporal Bayesian Modeling Using R". Volume 63: Journal of Statistical Software, January 2015.
[12] Gelman A, Carlin JB, Stern HS, Rubin DB (2004). Bayesian Data Analysis. 2nd edition. Chapman & Hall/CRC, Boca Raton.
[13] Gelfand AE, Ghosh SK (1998). \Model Choice: A Minimum Posterior Predictive Loss Approach." Biometrika, 85, 1-11.
[14] Banerjee S, Carlin B P, and Gelfand A E 2004 *Hierarchical modeling and analysis for spatialdata*(Boca Raton: Chapman & Hall/CRC).
[15] K. SHUVO BAKAR, PHILIP KOKIC, " Bayesiangaussian models for point referenced spatial and spatio-temporal data". Vol. 51, No. 1, pp. 17-40, Journal of Statistical Research, 2017.
[16] Bakar KS (2012). Bayesian Analysis of Daily Maximum Ozone Levels. PhD Thesis, University of Southampton, Southampton.