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Bio-Inspired Design Tool to Emerge Multifractal Forms

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ABSTRACT: *Complexity has been long defined as a general law of nature based on the sudden emergence of new organizations. Langton, in his seminal paper of 1992 explained the general idea behind emergence that shows local variables interacting together at the object-level, and the sudden emergence of global structure above it, which then feeds back on the local variables. Examples of this can be found in all living organisms and even super organisms, such as a beehive, city or any type of social organization. This interacting capacity for achieving balance between ordered and chaotic systems, interprets the spreading of this approach into various scientific fields, especially at the efficient design and bio-inspired geometrical development. In Multifractal geometries the value of urban connectivity and complexity threshold forms a phase transition "sudden emergence" of the dynamical urban behaviour that guarantees the complex shapes continuity's, homogeneity's and coherence's. The research will utilize a new bio-Inspired design tool (BIDT), (to emerge adaptive Multifractal forms), which possess a dynamic capability of deterministic chaos behaviour mixture, and based on weighted fitness function design to achieve hybrid control of the optimization criterion between the designer and evolutionary process. The focus of this research is on emerging Multifractal forms that adaptively self-organized depending on the feedback, through BIDT for evolutionary multi-criteria optimization. The research will address the morphological challenge that aimed to transcend the building complexity and urban connectivity thresholds. This morphological approach produce an objective method for assessment urban connectivity and complexity, which helps respond to the modern and efficiency geometric computing challenges.*

Keywords *- Adaptive Multifractal forms, multiple design challenges, weighted fitness function design*

1. INTRODUCTION

Natural selection and evolution develop a huge amount of fascinating biological form-finding mechanisms in different environments (e.g. beehive, spider and silkworm), which possess many inspiring properties, motivate scientists and engineers to find some useful clues to create new complex man-made or update the existing ones. These biological form-finding mechanisms characterized by a powerful capacity to create infinite geometrical constructions, stemmed from a systematic form of interaction, where an intimate correlation of local agency and environmental stimulus constructs collective (adaptive) orders allows for the emergence of new organizations [1]. According to Jencks, complexity theory is based on the sudden emergence of new organizations, even if it cannot fully explain the miraculous power behind it [2]. Chris Langton (1992) has explained the general idea behind this concept, in his emergence diagram, 1992, that shows local variables interacting together at the object-level, and the sudden emergence of global structure above it, which then feeds back on the local variables. According to Gershenson (2008), a complex system is one in which elements interact and affect each other's so that it is difficult to separate the behaviour of individual elements [3]. This interacting capacity for achieving balance between ordered and chaotic systems, interprets the spreading of this approach into various scientific fields, especially at the efficient design and bio-inspired geometrical development. The growth of a city took hundreds or even thousands of years', If we consider the growth of a city as a self-organizing biological-like organism, organic urban forms will take hundreds or even thousands of years to emerge and evolve till the current morphology, co-evolving with the human bio-functional interrelated evolution. In particular, this urban forms will show an inside fractal nature [4].

Michael Batty, (1994) was the first who proved the fact that fractal dimension is a valid descriptor and classifier of urban growth [5]. His approach has been followed by many researcher of the field, who performed serious of analytical studies (Frankhauser 1998, Jeffrey West 1999, Salingaros 2001, Alexander 2004, McAdams 2007, 2009, Tucek 2013, Swaid 2015), thus demonstrating the capacity of lacunarity and fractal analyses to measure the complex shapes and describe the degree of hierarchy, interactions, connectivity and complexity. Tucek & Janoska (2013), performing a significant study on the same topic, showed how spatial processes can be described in terms of the fractal dimension and how those descriptors can be used to view the city not as a series of different states in time, but as a changing system with its own dynamics. If biological forms uses this adaptive mechanism of emergence and if the emerging forms have a fractal nature, can we, in turn, achieve a reverse engineering process in order to re-create this way of proceeding in the evolution of architectural forms? Trying to synthetize the ability of evolution to create complex self-similar structures from simple rules, an innovative mechanism has been developed to analyse and generate complex urban forms by adaptive architecture geometries corresponding with two interrelated relations:

A. A bio-inspired space-filling mechanism (BSM),for the emergence of adaptive Multifractal forms, responding to multiple current design challenges (both morphological and social), thus maximizing the geometry needs of connectivity and the complexity value of biological inspired architectural forms. The mechanism of the space filling composing and shaping is based on a natural model about silkworm's Multifractal structure. Where achieving the value of urban connectivity and complexity threshold forms a phase transition "sudden emergence" of the dynamical urban behaviour, which guarantees the complex shapes' continuity, homogeneity and coherence.

B. A parametric design (PD) (written in python and combined with the visual programming of Grasshopper as a plug-in to "Rhinoceros 5" software) will be integrated with BSM, to establish parametric relations between modern geometric computing and multiple current design challenges, in order to translate the biological space-filling mechanism into the dominant Multifractal architectural configuration.

C. This translation process will be performed by Hypervolume estimation algorithm (HypE) for evolutionary multi-criteria optimization (EMO), which simply measures the volume of the space that is dominated by a solution set, bounded by a so-called reference point [6]. In particular, the research is needed to resolve the following questions:

1. How to achieve hybrid control of the optimization criterion between the designer and Evolutionary process?
2. What attributes of the biological model may be used for building self-organizing issues?
3. Why Multifractal design could permeate and adapt with the phenotype and genotype successive mutations?
4. How could be model, simulate and comprehensively identify the design of adaptive Multifractal forms inspired by biological space-filling mechanisms?

2. BACKGROUND

Whilst, morphological and social approaches have ever been played a significant role for un/digital building evolution of architecture efficient design, in the recent years. Vast of architectural approaches (functional, structural, biological design, material behaviour and environmental) which have applied as form finding optimization to emerge complex shapes and materials, omitted the need to tackle morphological and social challenges on one hand, and defining the problems very well on the other hand [7]. Thus, created critical obstacles at the generated buildings' level and at the modern geometric computing's level itself. Prompting Cedric Price to wonder if computing is the answer, but what was the question? These obstacles justify the incapability of these form-finding processes to sympathize between the complex morphology co-evolving with the human bio-functional interrelation evolution, due to neglect the social values as a determinant for an evolutionary design process. Which generate a pivotal question about the designer role in a digital evolutionary system?

Although, that John Frazer in his seminal book An Evolutionary Architecture (1980), presented a fundamental cybernetic thesis that 'architecture is a living, evolving thing', Gordon Pask, was who introduced the idea that 'architects are first and foremost system designers who have been forced to take an increasing interest in the organizational system properties of development, communication and control' [1]. Frazer elucidated the evolutionary model of nature as the generating process for architectural form, in an attempt to achieve in the built environment the symbiotic behaviour and metabolic balance that are characteristic of the natural environment [8]. Where he considered the architecture as a form of artificial life, subject, like the natural world, to principles of morphogenesis, genetic coding, replication and selection. Katherine Fu, considered understanding the cognitive mechanisms that underlie bio-inspired design, as well as developed tools and methods to support it, as constraints for finding elegant analogies (inspired form a natural) without being well versed in biology, and/or without counting on isolated experiences or chance [9]. In the field of nature, biological self-organization structure enables an organism to establish new connections and enhance the current complexity through morphological and social behaviour. An efficient example of this can be found in all living organisms and even super organisms, such as a beehive, city or any type of social organization [2]. In addition, many studies in the bio mimetic area have already shown that natural models can achieve highly efficient results [10].

3. METHODS

The research applied the following methodology to Cosenza city, which involved four consecutive stages: Geometry's connectivity and complexity analyses; selection and investigation of biological model; embodying bio-inspired methods into a software tool; and optimization algorithms.

3.1. Geometry's connectivity and complexity analyses

The research has developed two algorithms of fractal and lacunarity dimensions to analyse the current geometry's connectivity and complexity behaviour. Where the ability of lacunarity value to measure the heterogeneity in the urban structures represents one of the significant indicators for the urban connectivity dimension, in addition to the local connected fractal value.

3.1.1. Local Connected Fractal Dimension (LCFD): the first algorithm is responsible for measuring local connected fractal dimension, where the scan considers the pixel distribution in terms of local environments [11]. The computer program measured the total number of pixels local connected in a box of increasing size ε centred at a point x, y [12]. In this context, "local connected fractal dimension" relates to all the pixels within the largest box used for the analysis that belong to the cluster connected to the pixel on which the box is centred. The scaling relation is found by the linear regression of the logarithm of the mass in a box of size ε on the logarithm of ε. The relationship is expressed **as $\alpha = \log [M(\varepsilon)] / \log(\varepsilon)$.** (1) Where M (ε) is the number of local connected points (eight-neighbourhood connection) in a box of side size ε. The Algorithm procedure is as follows: deconstruct all current geometries to point list. **Algorithm 1** was written by python and returns the number of box coverage of point list for boxes of size scale. In local connected dimension box counting algorithms, the box for each scale deviation is centred on each point of interest and scaled into three frequency distributions (FQ). The N is the number of divisions made on each dimension. Then calculates the Local Connected Fractal Dimension (LCFD) of point list between box scales scale1, scale2 and scale3 via the box-counting algorithm.

Algorithm 1: Local connected fractal dimension

Require: Point list, Box count Scale
1: **Procedure** Call the current Point list.
2: N = 1/ ε # Assign the Number of divisions made on each dimension
3: FQ1 = N / 4 # frequency distribution 1
4: FQ2 = N / 2 # frequency distribution 2
5: FQ3 = 3*N / 4 # frequency distribution 3
6: Find all the points in the scaled box to P within 4 scaled boxes centred at P (this is the "local connected set" S).
3: Count the number of points M (ε) of S in boxes of decreasing side size ε ($0.01 \le \varepsilon \le 0.03$) centred at P.
4: **Return** Calculate the local connected fractal dimension of S relative to P using equation 1 by linear regression
of **log (M (ε)) versus log (ε).**
5: **end Procedure**

3.1.2. Sliding Lacunarity Analysis (SLAC): the second algorithm is responsible for measuring the Lacunarity of the current structure. Lacunarity analysis is a multi-scaled method for characterizing the morphology of urban spatial patterns across different scales [13]. In addition, for describing patterns (has the ability to describe binary and quantitative data) of spatial dispersion [14]. In addition, lacunarity analysis allows the determination of scale dependent changes in spatial structure, and reveals the presence and range of self-similarity. These abilities achieve one of the significant goals in architecture and urban design is the quantification of spatial patterns. In this paper lacunarity, analysis will investigate describing the spatial gaps' distribution of the urban structure in the case study incorporated with fractal parameter. According to Gefen, Meir, and Aharony Lacunarity definition is the deviation of a fractal from transitional invariance. Transitional invariance is highly scaled dependent; sets, which are heterogeneous at small scales, can be quite homogeneous when examined at larger scales or vice versa [14]. Thus, lacunarity measures heterogeneity to complement the fractal dimension in describing complexity.

Gliding-box algorithm: The logarithm, which had been used in this paper for calculating lacunarity, is the gliding-box algorithm, which was proposed by Allain and Cloitre (1991). According to Fliho (2008), this algorithm has a box of size r slides over an image. The number of gliding-box with radius r and mass M is defined as n (M, r). The probability distribution Q (M, r) is obtained by dividing n (M, r) by the total number of boxes. Lacunarity at scale r is defined as the mean-square deviation of the variation of mass distribution probability Q (M, r), divided by its square mean [13].
$L(r) = \sum m\ M2Q\ (M, r) / [\sum m\ M2Q\ (M, r)]^{2}$ (2). Where: L(r) = lacunarity at box size r. M = mass or pixels of interest, Q (M, r) = probability of M in box size r. **Algorithm 2** was written by python and returns the number of box coverage of point list for boxes of size scale with interval N/4 with changing levels of magnification, N is the number of divisions made on each dimension. Then calculates the lacunarity dimension of point list between box scales via the sliding box-counting algorithm, and average and standard deviation have solved by grasshopper components.

Algorithm 2: Lacunarity analysis

Require: Point list, box count scale
Procedure Call the current point list.
N = []
Scale1 = 3 / 4
Scale2 = 2 / 4
Scale3 = 1 / 4
Step1 = 8400 / 4
Step2 = 2 * scale1 /40
Step3 = 3 * scale1 /40
Find all the points in the sliding box to P within 3-scaled Boxes centred at P and overlapped by steps 1, 2 and 3.
Count the number of points M (ε), in sliding boxes of Side size ε (0.25 ≤ ε ≤ 0.75) centred at P.
For every box scale calculate the box variance by (((box counts[i] - mean) ** 2) ** 0.5) ** 2, then calculate the average variance by divide total variance to number of scales.
Return calculate lacunarity dimension of point list P using equation 2 by (variance / (mean **2)) + 1
end Procedure

The research has generated the city map by "ELK" technology[1], and then has threshold urban interactions, by multivariate linear discriminant function analysis based on the values of local connected fractal dimensions and lacunarity analyses. This invaluable local dimension analysis forms the core for estimating localized social, morphological changes of different urban interactions levels, as a base of fitness function design for Multifractal forms evolution [15].

3.2. Selection and investigation of biological model

Based on the significant study by Oxman (2013), the research adopted a natural model of hierarchical space-filling mechanism, which refer to specific silkworm's shape-generation process and the spinning behaviour of silkworm in this study [16]. In particular discoveries on biology-driven strategy for shape-generation computation. Replacing such computational processes with biological ones allows informing shape-generation processes as well as spatial material organizations within a single (biological) system. As a result, natural systems typically exhibit high levels of integration between shape, structure and material making nature's designs highly efficient and effective forms of "computation" [16]. The main discovery showed that spinning configurations and fibre density distribution would vary according to the morphological features of the "hosting environment". Where the silkworm's filing-space mechanism has effectively adapted with the relative number and location of the available connections (vertical axis), see Fig 1. Which, depict four diverse cases of silkworm's hierarchical space-filling mechanism.

[1] Combined with visual programming of grasshopper as a plug-in to "Rhinoceros 5" software. Where data of streets, railways. Waterways, buildings, and various facilities maps collected from OpenStreetMap.org and shuttle radar topography mission (SRTM) data from NASA/Jet propulsion laboratory, and were fed to the mapping component OSM location to generate digital maps "baked" in Rhinoceros multilayers' [17].

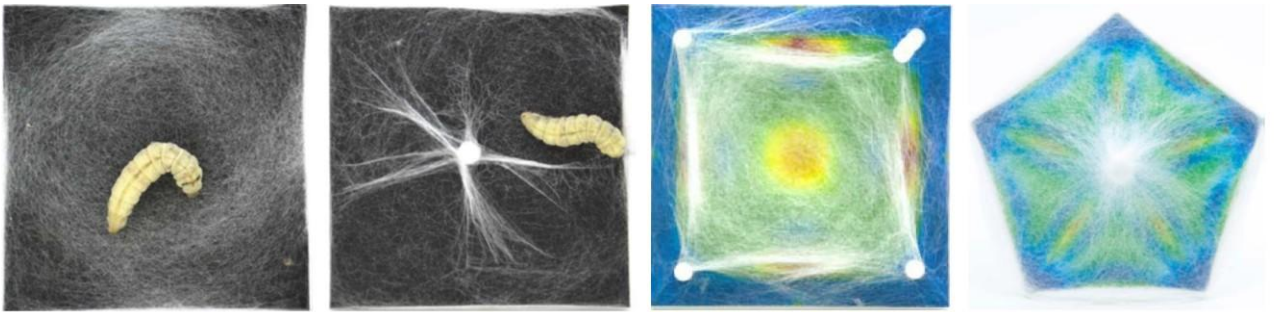

Figure 1: Four diverse cases of silkworm's hierarchical space-filling mechanism (Oxman, et al. 2013)

In the first case, due to the absence of a vertical axis (connection) the silkworm will spin a flat silk patch. In the second case, the variation in fibre density and organization reflect the morphological constrains given by the environment, also its worth to focus on the general correlation between the fibre density, where typically varied as a function of the distance from the central vertical axes. This case illustrated higher density deposition along the shortest distances from the geometrical centre to the surface boundary contour. The third case, possess four available connections showed Denser fibres appeared between poles along boundaries. The fourth case, showed greater density of silk deposition in areas of higher stress-strain.

3.3. Embodying bio-inspired methods into a software tool

In order to model and emerge adaptive Multifractal architecture, the research has developed a modelling software tool called Morphogenetic Fractal Architecture (MoFA). This tool constitutes from the incorporation between new Adaptive Dynamical Model ADM[2] for Parametric design (written in Python and combined with visual programming of Grasshopper as a plug-into "Rhinoceros 5" platform [17]. Which allow users and external application developers to integrate their applications) with HypE algorithm for EMO, which possess a dynamic capability of deterministic chaos behaviour mixture and based on weighted fitness function design. The building model established on this platform had multi-parameters including number and location of available connections, number of attractors, genetic diversity and user defined parameters. Whilst first parameter control the geometry connectivity (social) value; second parameter control geometry complexity (morphological) value, see Fig 2. These parameters form the core of weighted fitness function design, which guarantee a hybrid control of the optimization criterion between the designer and Evolutionary process. The process of the modelling testified translation the natural silkworm space-filling mechanism into BSM with Multifractal geometries, see Fig 3. The generated geometry showed interacting capacity with various connections, proportional to their distance from the central attractors, which in turn, their field effect influences are proportional to each attractor proximity to the connection (movement flows resources), as shown in Fig 4. According to this biomimetic, objective, Multifractal forms have been generated by controlling the correlated spatial and functional relationship characterized by high sensitive adaptability, through capacity of responsiveness to different environment situations and changes by mutating their structure, behaviour and function. The MoFA allows users to modify the values of the first and second parameters, and then generates different scenarios of Multifractal forms.

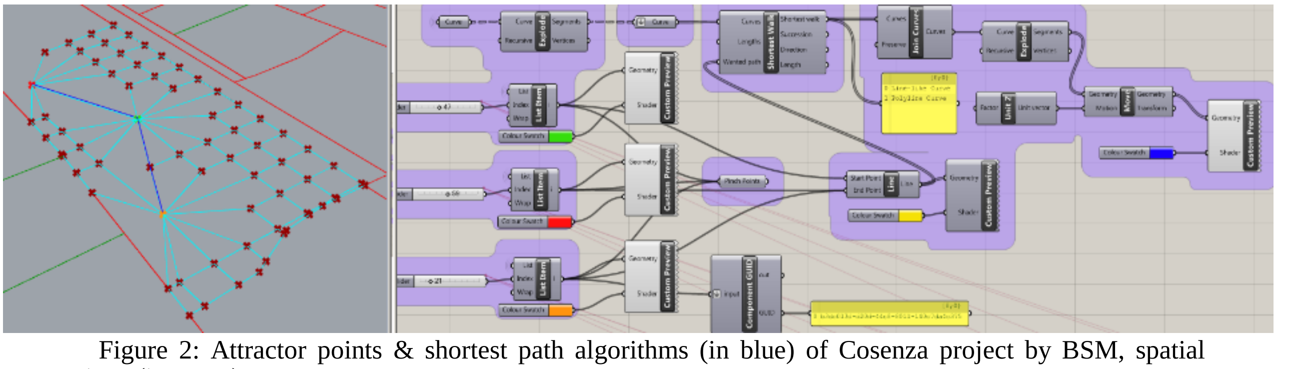

Figure 2: Attractor points & shortest path algorithms (in blue) of Cosenza project by BSM, spatial connections (in green).

[2] To model the optimization of building genotype and phenotype for seed emergence based on the local criteria (spatial emergence and spatial agency processes). In this context, a number of computational approaches (field effect, point attractor and power law scaling models') for modelling morphogenesis are compiling to study an integration.

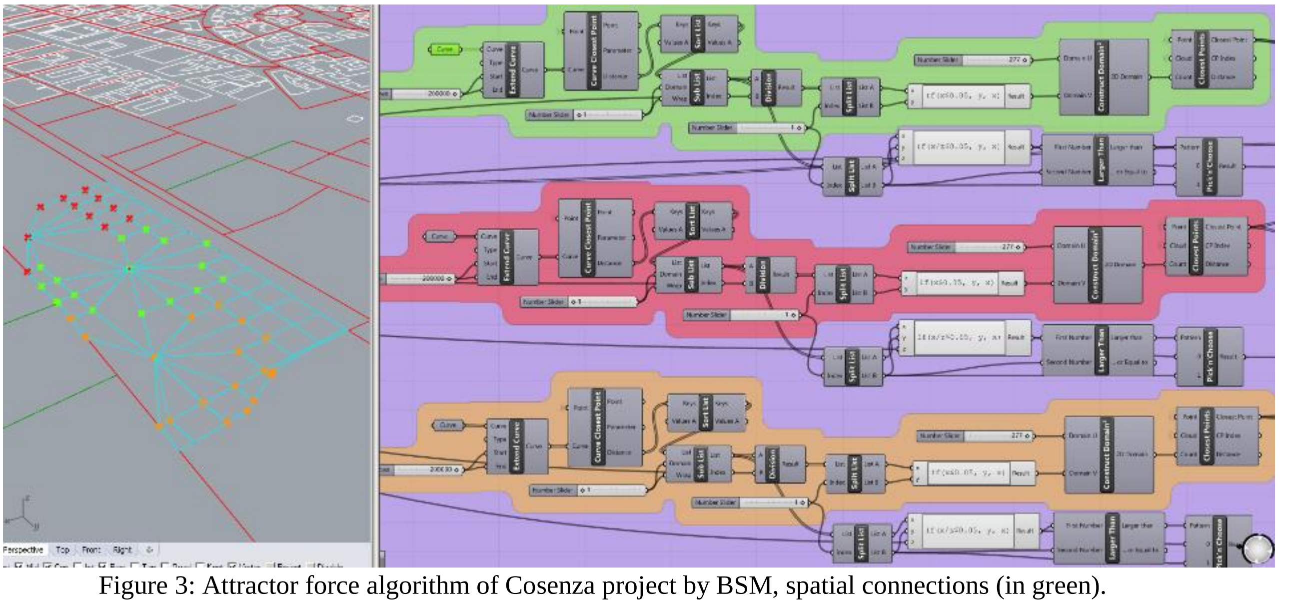

Figure 3: Attractor force algorithm of Cosenza project by BSM, spatial connections (in green).

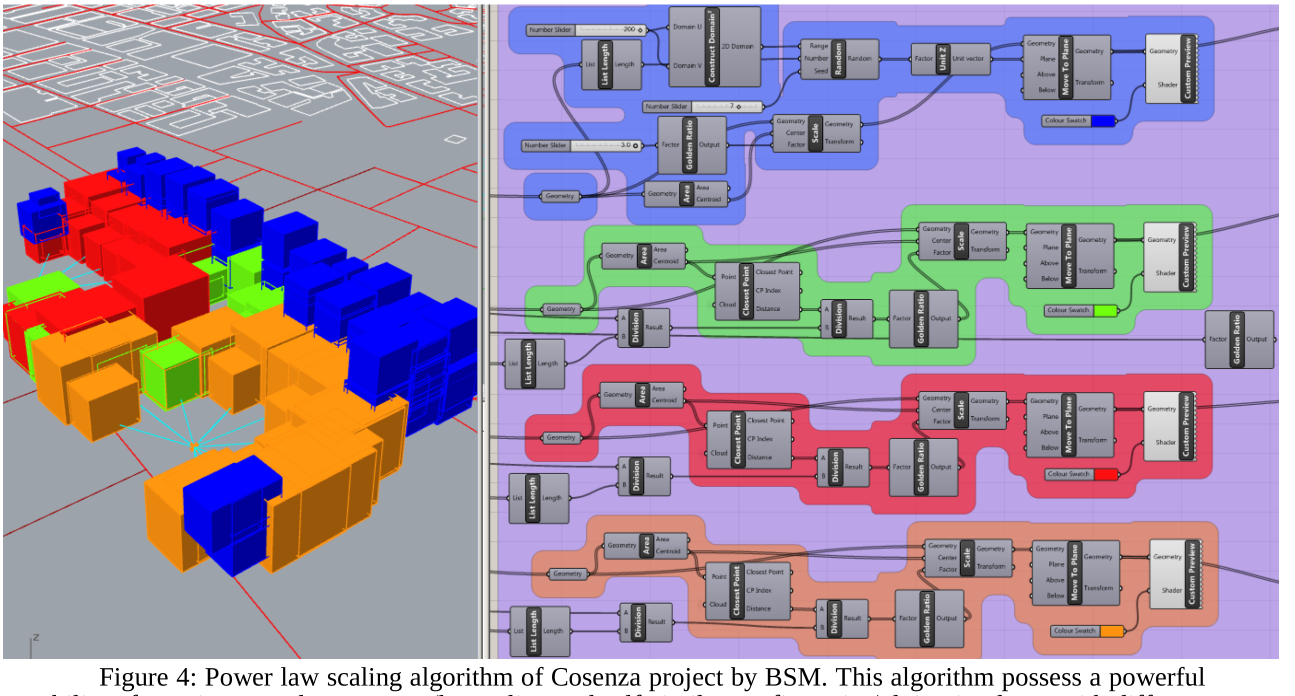

Figure 4: Power law scaling algorithm of Cosenza project by BSM. This algorithm possess a powerful capability of creating complex patterns (by scaling and self-similar configuration) lying in planes with different orientations in a multidimensional space and subject to the approach power law scaling model.

3.4. Optimization algorithms

The research adopted a quality indicator called Hypervolume indicator, in order to define the optimization goal for the multi-objective problem. This indicator assigns each Pareto set approximation a real value reflecting its quality and therefore can be used as objective function for the underlying set problem [18]. To analyse the results from a multi-objective parameter optimization, the research adopted a quantitative metrics for comparison of the Pareto sets called Hypervolume Estimation Algorithm (HypE) for Multi-objective Optimization, to choose one set for analysis from those found by multiple run [19]. The optimization algorithm was generated by Octopus[3], combined with visual programming of grasshopper, and is responsible for applying evolutionary principles to parametric design and problem solving. It introduces multiple fitness values to the optimization (In total 3 genes or design variables were manipulated by maximizing both genetic diversity, generated form's LCFD value, and minimizing SLAC value, see Fig 5). The best trade-offs between those

[3] Octopus is a plug-in for grasshopper, which implements two multi-objective evolutionary algorithms: SPEA-2 in its original form and HypE from ETH Zürich [22].

objectives are searched, producing a set of possible optimum solutions that ideally reach from one extreme trade-off to the other [20]. The research has chosen the strategy of HypE reduction' of how a Pareto non-dominated front should be truncated to fit the archive size when it is too big, also choose HypE mutation strategy. Table 1, showing the settings are used during the optimization for Weighted Fitness Process.

Table 1: Main setting for the optimization process

Population size:	50
Generations:	5
Maximum calculation time:	1s
Elitism:	0.5
Mutation probability:	0.1
Crossover rate:	0.8
Mutation rate:	0.5

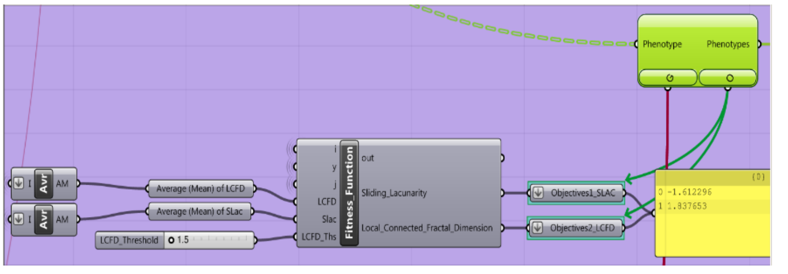

Figure 5: Weighted fitness function algorithm and octopus solver

4. RESULTS AND REFLECTION

The task was to optimize the local connected fractal dimension, lacunarity dimensions and genetic diversity by HypE for the evolved geometries, through manipulating the genetic code (where the parameterization comprises the spatial location of the attractors). The research has applied weighted fitness function model to optimize the evolved geometries that only achieve the desired threshold, see Fig 6 (left sideshows the geometries after the weighted fitness function algorithm and right side shows the evolved geometries after genetic algorithms optimization reduced by HypE). Octopus facilitates visualization of the solution space by introducing a colour-range from red to green for the third dimension. Green hereby depicts the lower bound of the non-dominated set of solutions, red the upper bound [21], as shown in Fig 6. From the statistic data of weighted fitness function optimization, it can be seen that after five generations the final population of 50 individuals contains 25 dominated individuals and 25 non-dominated individuals. This, while having an archive size of 20, is one basic indicator of convergent behaviour tendencies.

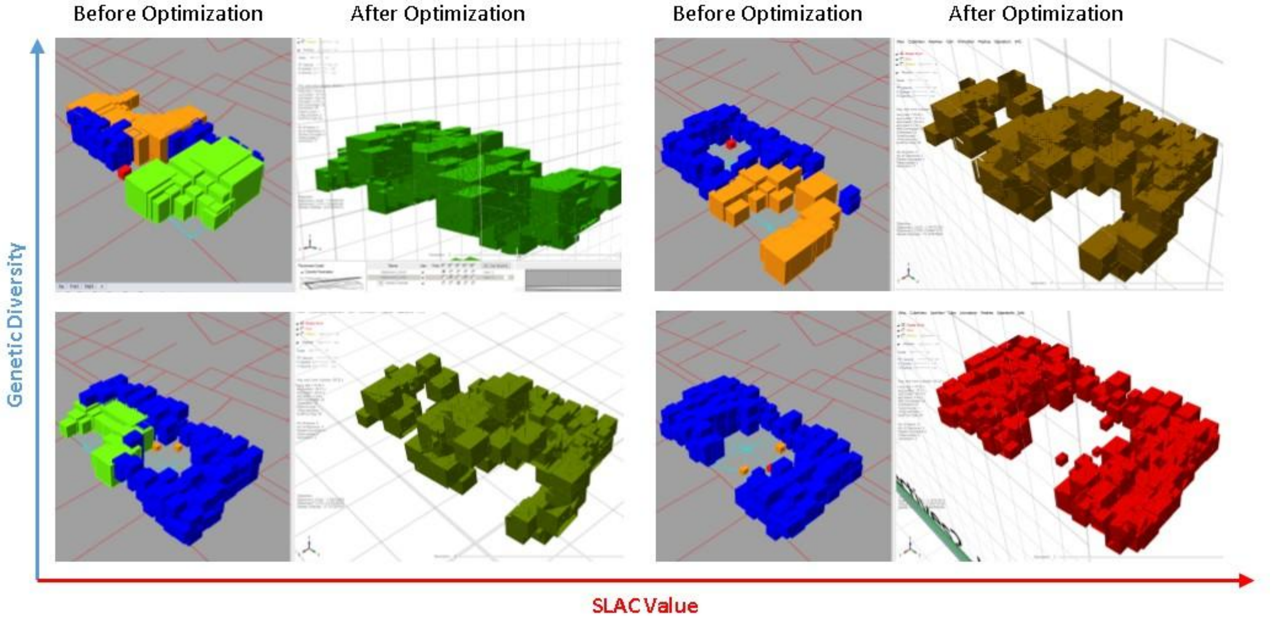

Figure 6: Some results of weighted case search process after 5 generations, population size 50, and mutation rate 0.5

In summary, whilst the green weighted cases (groups) of high urban connectivity and complexity dimension possess a high stability of urban network morphology (spatial homogeneity = low Lacunarity dimension), and the red weighted cases (groups) of lower urban complexity and connectivity dimension possess a low stability of urban morphology (spatial heterogeneity = high Lacunarity dimension).

Figure 7: Artificial selection (by designer) of one of the Pareto non-dominated front, based on non-quantifiable criteria

Utilizing computational optimization demonstrate high level of design enrichment, as shown in Fig 7. This research shown the efficiency of EMO in engineering design, which opens the door toward achieving a holistic and integrative design approach by incorporating more objectives (functional, environmental, material behaviour and biological design).

5. CONCLUSION

Enhancing the understanding of architecture through the principles of generation, fabrication and space require investigation the notion of self-assembly and self-organizing as a catalyst for architectural provocation and investigation. From John Frazer's "An evolutionary architecture", Greg Lynn's "Animate Form" to contemporary fascinations with biomimetic, bionic and bio-inspired design principles, and the architectural discourse has long been inspired by the power of nature. This research has shown how the silkworm may be considered as an autonomous agent in processes of design optimization. Due to the hierarchical silkworm's shape-generation process and the spinning behaviour of silkworm, which represent a crucial attributes may be used for building self-organizing issues. In addition, the research proved how self-organizing change forms a fundamental character of new language of form based on Multifractal design, chaos theory, nonlinear dynamics and complexity theory itself, and allows permeating and adapting with the phenotype and genotype successive mutations.

The major challenge in this context is the integration of user preferences to direct the search. Therefore, the research has developed weighted fitness function algorithm. This weighted fitness function algorithm aims to provide flexibility with respect to the search direction that the Hypervolume indicator formalize, by adjusting the optimization goal according to the designer vision for the future connectivity and complexity value (which reflect the social and morphological approaches). The latter aspect opens new perspectives in joining interactive approaches in the field of multiple criteria decision making with the set-based approach pursued in the EMO field [18]. This aspect according to Brownlee, 2012, forms a critical condition to realize the benefit of the optimization process if only the results of the optimization can be analysed in a way that aids the decision-making process and the selection of the final design solution [21]. Which allows social values to be used as determinant for an evolutionary design process, and catalyst the designer to play powerful role in a digital evolutionary design for achieving hybrid control of the optimization criterion between the designer and evolutionary process.

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